Using Predicate Logic

Chapter 5

Using Propositional Logic

Representing simple facts

It is raining
RAINING

It is sunny
SUNNY

It is windy
WINDY

If it is raining, then it is not sunny
$RAINING \rightarrow \neg SUNNY$

Using Propositional Logic

- Theorem proving is decidable
- Cannot represent objects and quantification

Using Predicate Logic

- Can represent objects and quantification
- Theorem proving is semi-decidable

Using Predicate Logic

1. Marcus was a man.
2. Marcus was a Pompeian.
3. All Pompeians were Romans.
4. Caesar was a ruler.
5. All Pompeians were either loyal to Caesar or hated him.
6. Every one is loyal to someone.
7. People only try to assassinate rulers they are not loyal to.
8. Marcus tried to assassinate Caesar.

Using Predicate Logic

1. Marcus was a man.

   man(Marcus)

Using Predicate Logic

2. Marcus was a Pompeian.

   Pompeian(Marcus)

Using Predicate Logic

3. All Pompeians were Romans.

   $\forall x$: Pompeian(x) $\rightarrow$ Roman(x)

Using Predicate Logic

4. Caesar was a ruler.

   ruler(Caesar)

Using Predicate Logic

5. All Pompeians were either loyal to Caesar or hated him.

   inclusive-or

   $\forall x$: Roman(x) $\rightarrow$ loyalto(x, Caesar) $\vee$ hate(x, Caesar)

   exclusive-or

   $\forall x$: Roman(x) $\rightarrow$ (loyalto(x, Caesar) $\wedge \neg$hate(x, Caesar)) $\vee$

   ($\neg$loyalto(x, Caesar) $\wedge$ hate(x, Caesar))

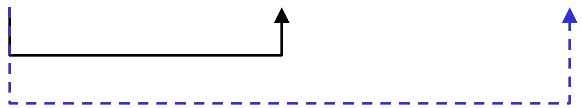
Using Predicate Logic

6. Every one is loyal to someone.

   $\forall x: \exists y: loyalto(x, y)$ $\exists y: \forall x: loyalto(x, y)$

Using Predicate Logic

7. People only try to assassinate rulers they are not loyal to.



$\forall x: \forall y: person(x) \wedge ruler(y) \wedge tryassassinate(x, y)$

$\rightarrow \neg loyalto(x, y)$

Using Predicate Logic

8. Marcus tried to assassinate Caesar.

   tryassassinate(Marcus, Caesar)

Using Predicate Logic

Was Marcus loyal to Caesar?

man(Marcus)

ruler(Caesar)

tryassassinate(Marcus, Caesar)

$\Downarrow$ $\forall$x: man(x) $\rightarrow$ person(x)

$\neg$loyalto(Marcus, Caesar)

Using Predicate Logic

- Many English sentences are ambiguous.
- There is often a choice of how to represent knowledge.
- Obvious information may be necessary for reasoning
- We may not know in advance which statements to deduce (P or $\neg$P).

Reasoning

1. Marcus was a Pompeian.
2. All Pompeians died when the volcano erupted in 79 A.D.
3. It is now 2008 A.D.

Is Marcus alive?

Reasoning

1. Marcus was a Pompeian.

   Pompeian(Marcus)

2. All Pompeians died when the volcano erupted in 79 A.D.

   $\text{erupted(volcano, 79)} \land \forall x\text{: Pompeian}(x) \rightarrow \text{died}(x, 79)$

3. It is now 2008 A.D.

   now = 2008

Reasoning

1. Marcus was a Pompeian.

   Pompeian(Marcus)

2. All Pompeians died when the volcano erupted in 79 A.D.

   $erupted(volcano, 79) \wedge \forall x: Pompeian(x) \rightarrow died(x, 79)$

3. It is now 2008 A.D.

   now = 2008

$$\forall x: \forall t_1: \forall t_2: died(x, t_1) \wedge greater\text{-}than(t_2, t_1) \rightarrow dead(x, t_2)$$

Variables and Quantification

With connectives we can say things like "If Mithu is a parrot then Mithu is green". But if we want to say things like "If anything is a parrot, then it is green", we need to introduce predicate calculus variables and quantifiers.

There are two kinds of quantifiers.

- **Universal quantifiers :** Denoted by ∀ universal quantifiers say that something is true for all posssible values of a variable.

Using universal quantifiers we can express the sentence "All parrots are green".

$$\forall x \, [\text{ parrot } (x) \Rightarrow \text{ color } (x, \text{green}) \,]$$

Here, the formula being quantified is an implication and x is a quantified variable. More precisely it stated that for all objects, if object is a parrot then the color of the object is green.

- **Existential quantifier :** Denoted by ∃. The formula consisting of an existential quantifier has a value true for at least one assignment of x to an entity in the domain.

For e.g. " There is a person who drives the car" could be represented as

$$\exists x \text{ drives } (x, \text{car})$$

Examples

Ram is a man.

$man(Ram)$

God loves Ram.

$loves(God, Ram)$

Ram eats an apple.

$eats(Ram, apple)$

Sita drinks water.

$drinks(sita, water)$

Every man loves god.

$\forall x\ (x, man(x) \dashrightarrow loves(x, god))$

Nobody likes taxes

$\neg (\exists x)\ likes(x,taxes)$

Everyone loves someone

$\forall x \exists y\ Loves(x,y)$

There is one person everyone loves

$\exists y \forall x\ Loves(x,y)$

All birds have feathers.

$\forall x\ [Feathers(x) \Rightarrow Bird(x)]$

Every dog is an animal.

$\forall x\ (dog(x) \Rightarrow animal(x))$

Every boy has a bicycle.

$\forall x\ (\exists y\ (boy(x) \Rightarrow (bicycle(x)\ own(x,y))))$

Some Predicate Calculus Equivalences

Suppose p and q are predicates; X and Y are variables

- $\neg \exists X \ p(X) = \forall X \neg p(X)$
- $\neg \forall X \ p(X) = \exists X \ \neg p(X)$
- $\exists X \ p(X) = \exists Y \ p(Y)$
- $\forall X \ p(X) = \forall Y \ p(Y)$
- $X \ (p(X) \wedge q(X)) = \forall X \ p(X) \wedge \forall Y \ q(Y)$
- $\exists X \ (p(X) \wedge q(X)) = \exists X \ p(X) \wedge \forall Y \ q(Y)$

RULES OF INFERENCE

In predicate calculus, there are rules of inference, that can be applied to certain wff and set of wffs to produce new wffs. Those rules are:

1. ***Modus ponens.*** This is the operation which produces the wff W2 from the wffs

   W1 and
   $W1 \rightarrow W2$

2. ***Modus tollens.*** This is the operation which produces $\neg W1$ from

   $\neg W2$ and
   $W1 \rightarrow W2$

3. ***Universal specialization.*** This produces W(A) from the wff $\forall x W(x)$, where A is a constant symbol.
   Using the above rules, we can produce W2(A) from wffs

   W1(A) and
   $\forall x\ W1(x) \rightarrow W2(x)$

   For example, consider the assertion "Leo is a lion" and implication "All lions are ferocious", we can derive the conclusion "Leo is ferocious".

   W1 : LION (leo)
   W2 : $\forall x$ LION(x) $\rightarrow$ FEROCIOUS(x)
   FEROCIOUS(leo)

   From W1 and W2, we have to show that FEROCIOUS(leo) is a logical consequence of W1 and W2.

   W1 $\wedge$ W2 : LION(leo) $\wedge$ ($\forall x$ LION(x) $\rightarrow$ FEROCIOUS(x))

   Since W1 and W2 are *true*, and LION(x) $\rightarrow$ FEROCIOUS(x) is *true* for all x, we can replace x by leo. So W2 will be $\neg$LION(leo) $\vee$ FEROCIOUS(leo), but $\neg$LION(leo) is *false*, so FEROCIOUS(leo) is *true*.

4. ***Chain rule.*** This is the operation which produces $P \rightarrow R$ from the wffs $P \rightarrow Q$ and $Q \rightarrow R$.