ELASTIC AND INELASTIC COLLISIONS

Linear Momentum and Collisions

The **linear momentum** of a particle of mass *m* and velocity *v* is defined as

$$\vec{p} = m \cdot \vec{v}$$

The linear momentum is a vector quantity.

It's direction is along **v**.

The components of the momentum of a particle:

$$p_x = m \cdot v_x \qquad p_y = m \cdot v_y \qquad p_z = m \cdot v_z$$

Elastic and Inelastic Collision

- We begin with a few definitions...

A collision in which the total kinetic energy is conserved is called an ***elastic collision***.

- What this means is $K_f = K_0$.

A collision in which the total kinetic energy is <u>not</u> conserved is called an ***inelastic collision***.

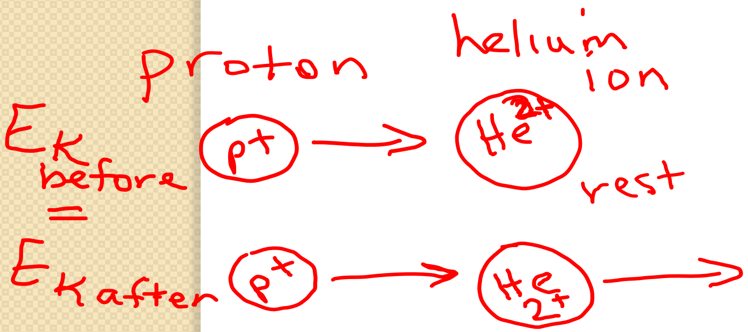
- What this means is $K_f \neq K_0$.
- Usually this means is $K_f < K_0$.

A collision in which the colliding objects <u>stick together</u> is called a ***totally inelastic collision***.

- What this means is that the final velocities are identical.

- Huygens' two types of collisions:

1. ***Elastic*** **Collisions** – a collision between masses where the kinetic before equals the kinetic energy after
   - objects cannot stick, crumble, bend or deform.
   - occurs with atoms or particles at subatomic level.

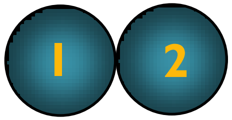


2. ***Inelastic*** **Collisions**
   - a collision where momentum is still conserved but kinetic energy is not conserved. $\sum E_{K_{before}} \neq \sum E_{K_{after}}$
   - some E_K is converted to sound, heat, mechanical energy, work.
   - objects can bend, stick, crumble, defor

*

Elastic collisions

- Suppose we have a perfectly elastic 1D collision between two equal masses, as shown:



- Mass 2 is originally stationary, and mass 1 is originally moving at 5 m/s.
- After the collision mass 2 is now moving to the right at 5 m/s, and mass 1 is stops.

Elastic Collisions

- After collision:
  - Two objects
    - Remain separate.
    - Remain same shape after collision.
  - Both momentum and KE remain the same

Lab system
&
Centre of Mass system

- In the **lab frame**, the situation looks like:↓

m_2 is initially at rest

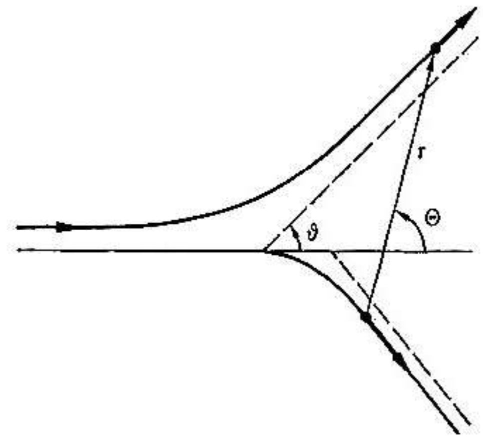


FIGURE 3.24 Scattering of two particles as viewed in the laboratory system.

- In the **CM frame,**

the situation looks like:↓

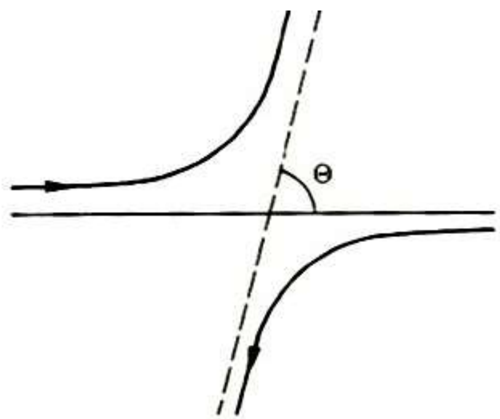


FIGURE 3.25 Scattering of two particles as viewed in the center of mass system

← Looks like this to an observer moving with the Center of Mass.

- In the **lab frame**: →

m_2 initially at rest. Connection between ϑ & Θ obtained by looking at detailed transform between lab & CM coordinates

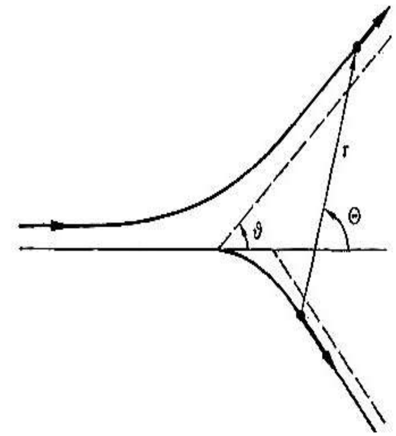


FIGURE 3.24 Scattering of two particles as viewed in the laboratory system.

In the **CM frame**: ↓

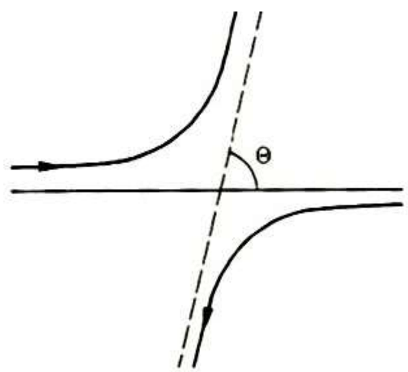


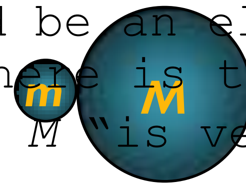
FIGURE 3.25 Scattering of two particles as viewed in the center of mass system

In the **CM frame,** the total linear momentum of the 2 particles **= 0.** Before scattering, the particles move directly towards each other. Afterwards, they move off as shown.

CM frame scattering angle Θ= same as scattering angle of either particle.

Elastic Collision

- Suppose we have a perfectly elastic 1D collision between two very unequal masses, as shown:
- The masses could be an elephant and a fly. The important thing here is that $M >> m$.



- $M >> m$ means that M "is very much greater than" m.
- Since $M >> m$ we can simplify the following:

$m + M = M$ $\quad$ $M - m = M$ $\quad$ $m / M = 0$

$M + m = M$ $\quad$ $m - M = {}^{-}M$

- Thus:

$$v_{1f} = \frac{2m_2 v_{2i} + (m_1 - m_2) v_{1i}}{m_1 + m_2} = \frac{2Mv_{2i} + (m-M) v_{1i}}{m + M} = \frac{2Mv_{2i} + {}^{-}Mv_{1i}}{M}$$

$$v_{mf} = 2v_{Mi} + {}^{-}v_{mi}$$

$$v_{2f} = \frac{2m_1 v_{1i} + (m_2 - m_1) v_{2i}}{m_2 + m_1} = \frac{2mv_{1i} + (M-m) v_{2i}}{M + m} = \frac{2m}{M} v_{1i} + \frac{M}{M} v_{2i}$$

$$v_{Mf} = v_{Mi}$$

Elastic collision of two particles

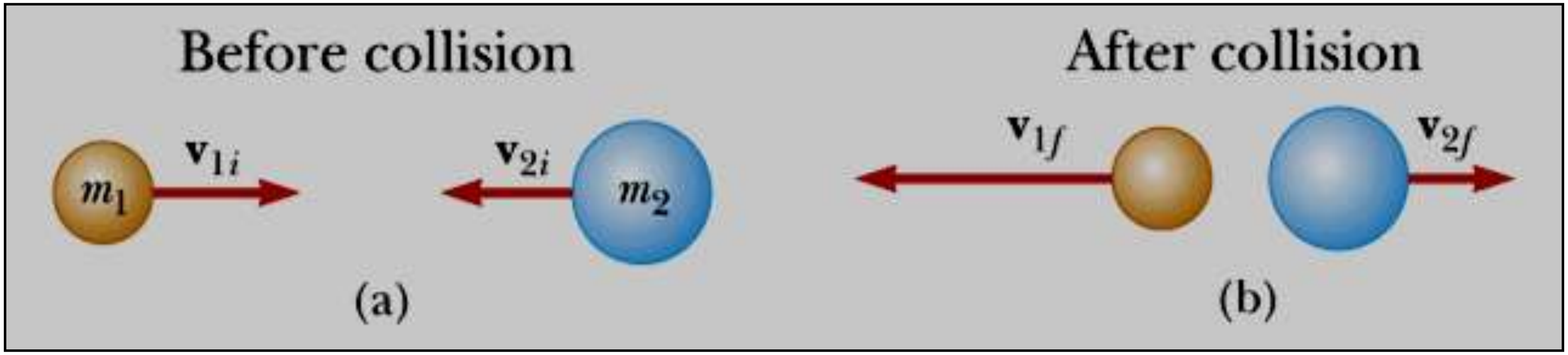
(Particles bounce off each other without loss of energy)

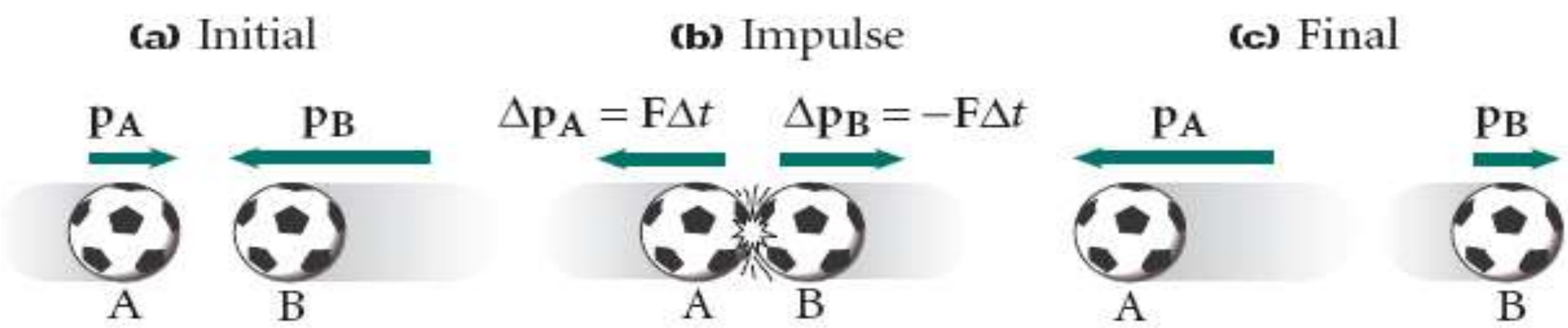
Momentum is conserved:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Energy is conserved:

$$\frac{1}{2}m_1 v_{1i}{}^2 + \frac{1}{2}m_2 v_{2i}{}^2 = \frac{1}{2}m_1 v_{1f}{}^2 + \frac{1}{2}m_2 v_{2f}{}^2$$





- KE is conserved in an elastic collision.
- If purely elastic both p and KE remain constant before, during, and after the collision.

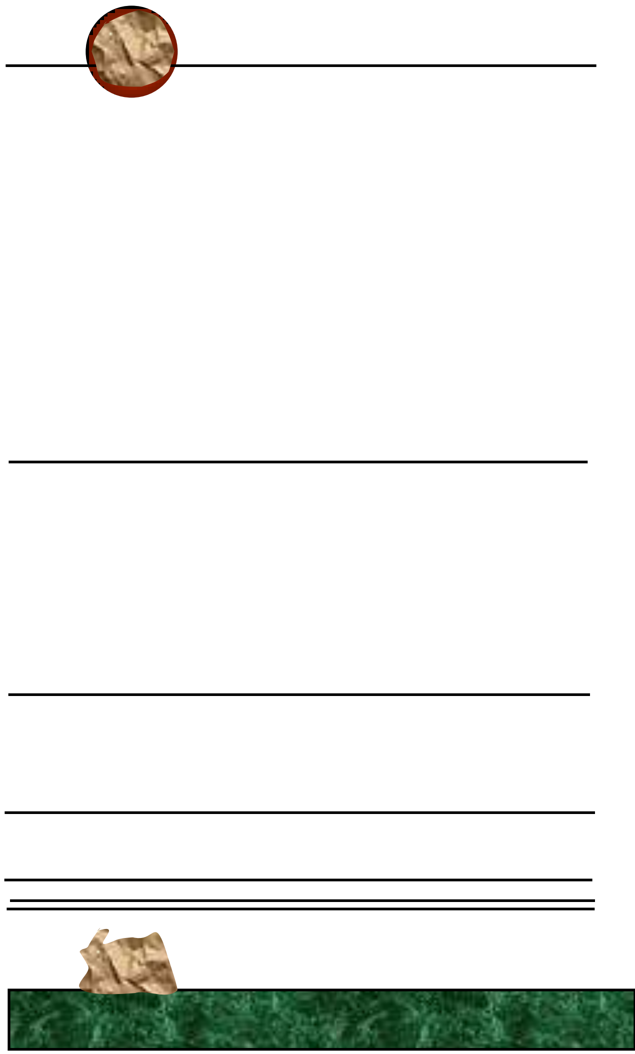
MOMENTUM AND KINETIC ENERGY REMAIN CONSTANT IN AN ELASTIC COLLISION

$$m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i} = m_1\mathbf{v}_{1,f} + m_2\mathbf{v}_{2,f}$$

$$\frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

Inelastic Collision

- An **inelastic collision** is a collision where energy is lost. The kinetic energy of the system is not conserved.
- For example, suppose we drop a billiard ball, as shown.
- Note that potential energy never regains its previous level.
- Each bounce loses a little more energy.
- We say that the collisions are **inelastic.**
- If instead of a billiard ball we drop a piece of clay, the clay will stick.
- We say that the collision is **completely inelastic.**



We define as **completely inelastic** any collision where the particles **stick together**.

Example of perfectly inelastic collision

Figure 6-10
When an arrow pierces a target and remains stuck in the target, the arrow and target have undergone a perfectly inelastic collision (assuming no debris is thrown out).

Perfectly inelastic collision of two particles

(Particles stick together)

$$\vec{p}_i = \vec{p}_f$$

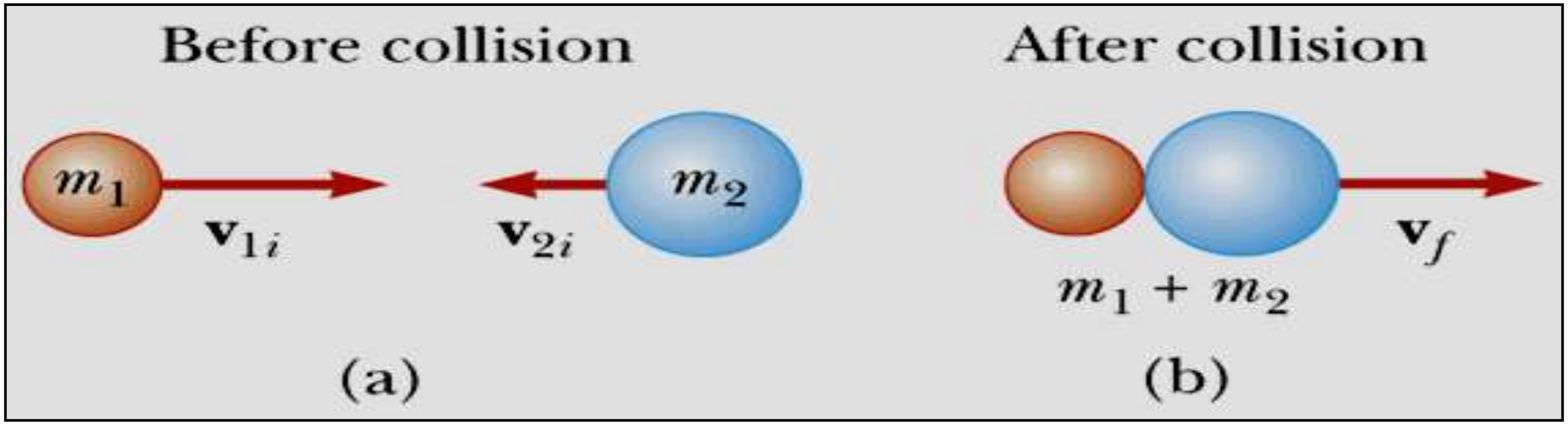
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

Notice that **p** and **v** are vectors and, thus have a direction (+/-)

$$K_i - E_{loss} = K_f$$

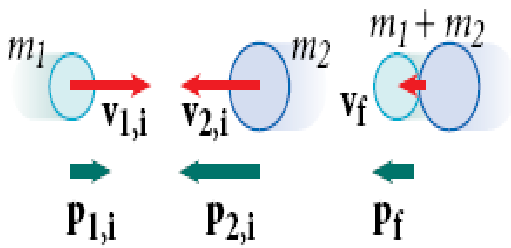
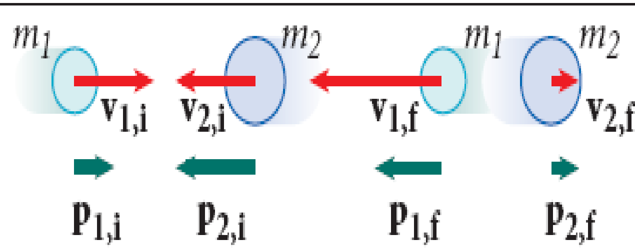
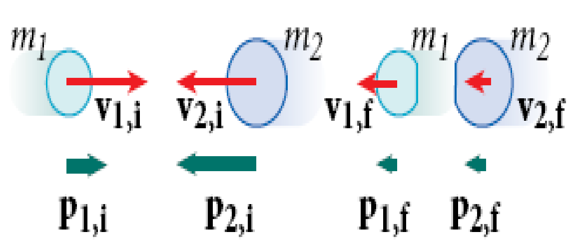
$$\frac{1}{2}m_1 v_{1i}^{\;2} + \frac{1}{2}m_2 v_{2i}^{\;2} = \frac{1}{2}(m_1 + m_2)v_f^{\;2} + E_{loss}$$

There is a loss in energy E_{loss}



Summary table of collisions

Table 6-2 Types of collisions

Type of collision	Diagram	What happens	Conserved quantity
perfectly inelastic		The two objects stick together after the collision so that their final velocities are the same.	momentum
elastic		The two objects bounce after the collision so that they move separately.	momentum kinetic energy
inelastic		The two objects deform during the collision so that the total kinetic energy decreases, but the objects move separately after the collision.	momentum

Some useful information

- Most collisions are neither elastic nor perfectly inelastic.
  - Most objects in an inelastic collision do not stick.
  - In near elastic collisions, objects lose energy to internal energy and sound.

Cross-section

The cross section of elastic scattering may be defined as the effective target area presented by the target to the incident beam of the particles during elastic scattering.

$$\sigma_{sc} \equiv (N_{sc}/I)$$

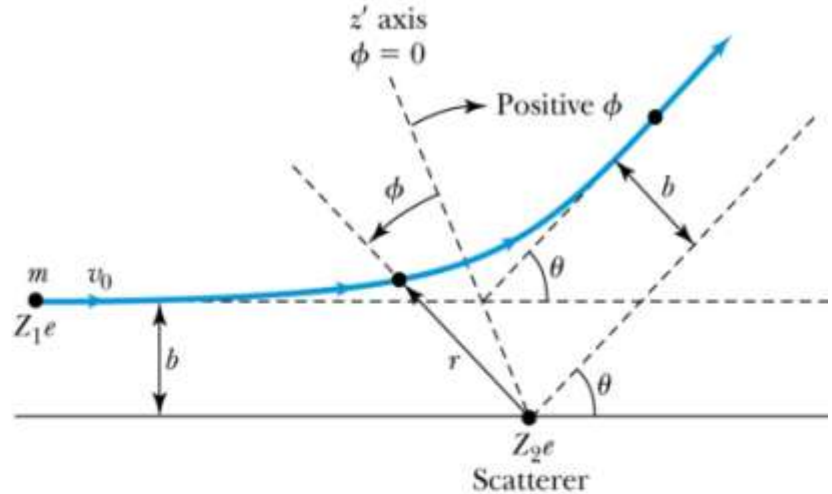
Where N_{sc} represents the number of particles scattered per target particle per second and 'I' represents the number of particles incident per unit area per second.

Unit: barn

Impact parameter

Impact parameter of an incident particle is defined as the perpendicular distance of the velocity vector of the particle from the centre of force.

There's a relationship between the **impact parameter** *b* and the **scattering angle** θ.



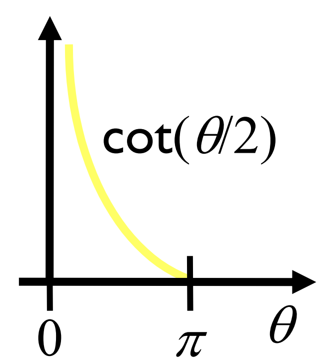
When *b* is small, *r* is small.

the Coulomb force is large.

θ can be large and the particle can be repelled backward.

$$b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K}\cot\frac{\theta}{2}$$

where $K = \frac{1}{2} m v_0^2$



Differential Cross Section of elastic scattering

$\sigma(\Omega)d\Omega \equiv (N_s/I)$

I = incident intensity

N_s = # particles/time scattered into angle $d\Omega$

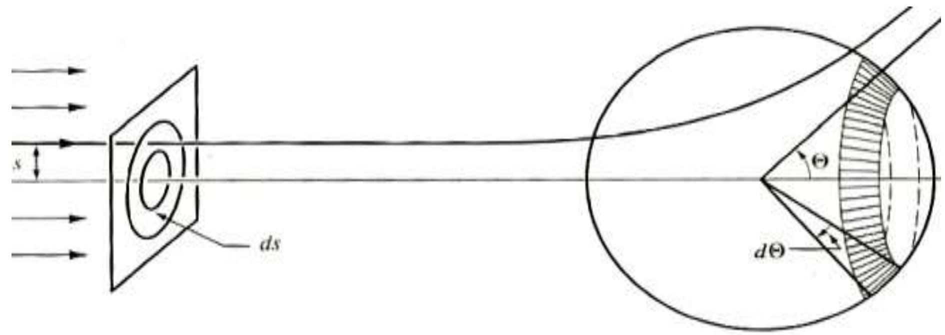


FIGURE 3.19 Scattering of an incident beam of particles by a center of force.

- In general, the solid angle Ω depends on the spherical angles Θ, Φ. However, for central forces, there must be **symmetry** about the axis of the incident beam

$\Rightarrow \sigma(\Omega)$ ($\equiv \sigma(\Theta)$) is independent of azimuthal angle Φ

$\Rightarrow d\Omega \equiv 2\pi \sin\Theta d\Theta$, $\sigma(\Omega)d\Omega \equiv 2\pi \sin\Theta d\Theta$,

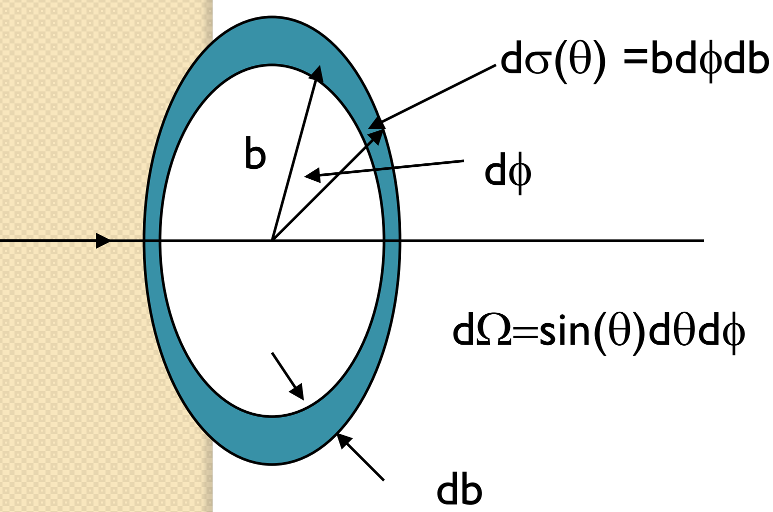
$\Theta \equiv$ Angle between incident & scattered beams, as in the figure.

$\sigma \equiv$ "***cross section***". It has units of area

Also called the differential cross section.

Differential cross-section in terms of impact parameter

- From these equations for θ we see that different values for the impact parameter correspond to different angles of scattering. Thus we can make a one-to-one correspondence between the scattering angle and b. In particular, we see that if particles approaching the target pass through an annulus of radius b and width db these will all emerge at an angle θ(b). This annulus area is the correct differential cross section for the angle of observation. We can invert the relationship to get b = b(θ), then db = |(db/dθ)dθ|. Consider the area of the section of the annulus shown.



$$\frac{d\sigma}{d\Omega} = \frac{b(\theta)d\phi db}{d\Omega} = \frac{b(\theta)d\phi}{d\Omega}\frac{db}{d\theta}d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{b(\theta)d\phi}{\sin(\theta)d\theta d\phi}\frac{db}{d\theta}d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{b(\theta)}{\sin(\theta)}\frac{db}{d\theta}$$

The Rutherford Scattering

$$\sigma(\Theta) = \left(\tfrac{1}{4}\right)\left[\frac{ZZ'e^2}{4\pi\varepsilon_0 E}\right]^2 \left(\frac{1}{\sin^4(\tfrac{1}{2}\Theta)}\right)$$

Eqn shows that no. of particles scattered in scattering is:

1) Directly proportional to the square of the nuclear charge Ze.

Focus of Hyperbola →

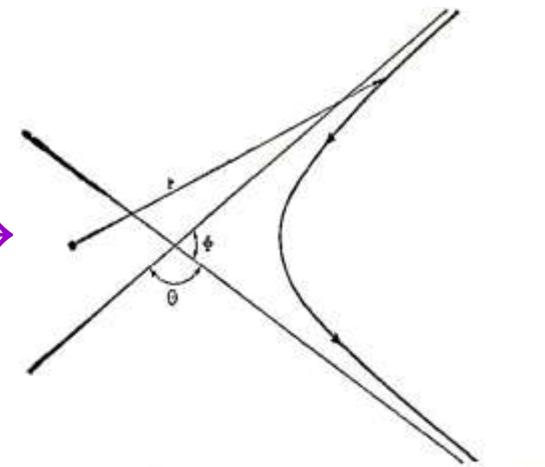


Fig. 3-15. Orbit for repulsive coulomb scattering, illustrating the connection between the angle between the asymptotes and the scattering angle.

2) Inversely prop to **$\sin^4(\tfrac{1}{2}\Theta)$**

3) Inversely prop to square of the initial kinetic energy E.