Coupled Oscillator

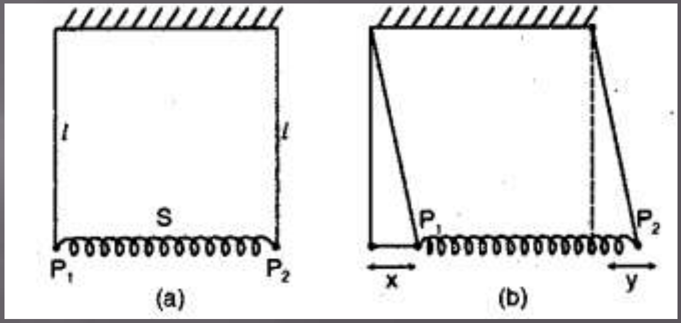
- Two or more oscillators linked together in such a way that an exchange of energy transfer takes place between them.
- In the coupled system, one of the oscillator may be source of energy or the energy may be given to one of the oscillators.
- A few examples of two coupled oscillator system:
  (a).Two simple pendulum with their bobs attached to each other by means of a string
  (b).The two coupled LC circuits.
  (c).Two masses attached to each other by three springs, middle spring provides the coupling.

Continued..

- Energy transfer takes place because two oscillators share a common component ,stiffness(capacitance) ,mass(inductance) or frictional force(resistance).
- These system have two degrees of freedom.
- In mechanical coupled oscillator the motion is completely specified by coupled mass than the displacement of bobs or the two masses are the required variables.
- In LC coupled circuits two variables are the currents in the circuits or the charges of two capacitors.

Two Stiffness Coupled Pendulums

- Two identical pendulums of same mass m suspended by a weightless rigid rod of length l ,connected by a spring of stiffness.



$$m\frac{d^2y}{dt^2} = \frac{-mgy}{l} + S(x-y)$$

$$m\frac{d^2x}{dt^2} = \frac{-mgx}{l} - S(x-y)$$

$$X = x + y$$

$$Y = x - y$$

- In phase mode of vibration
  when y=0 , x=y
- The equation describes the motion

$$\frac{d^2X}{dt^2} + \frac{g}{l}x = o$$

$$\frac{d^2Y}{dt^2} + \left(\frac{g}{l} + \frac{2s}{m}\right)x = o$$

$$w_0{}^2 = \frac{g}{l}$$

the stiffness does not play any role and both pendulums are always in phase.

Continued..

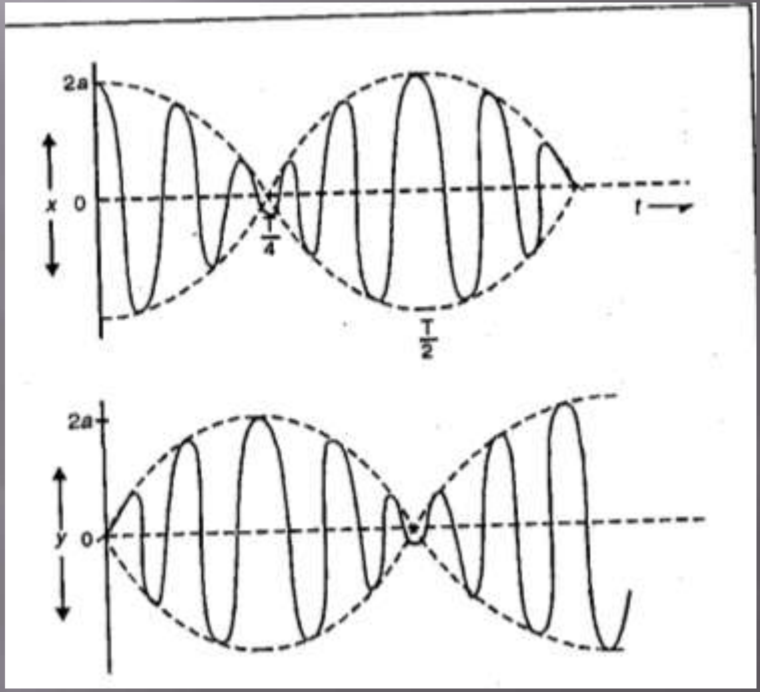
- Out of phase mode when x=0 , x=-y

  the motion is described by: $$w_2 = \sqrt{\frac{g}{s} + \frac{2s}{m}}$$

- Both the pendulums are always out of phase and frequency is more than the natural frequency of each oscillator.

Total Energy Of Coupled Oscillator

- Energy of pendulum A = $\frac{1}{2}\omega_a^2 A_1^2$
  Energy of pendulum B = $\frac{1}{2}\omega_a^2 A_2^2$
  Total energy = $2m\,a^2\omega_a^2$
- Total energy is constant ,the amplitude of the two pendulums is continuously varying with time.
- There is continuous exchange of energy between the two pendulums.


2a
x
0
T/4
T/2
t
2a
y
0

Inductive Coupling

- Inductively coupled are two ideal LC circuit with no ohmic resistance.
- Change of current in one circuit ,changes magnetic flux linked with it as a result induced emf is produced in both the circuit.

Continued..

- Coefficient of coupling $k = \frac{M}{\sqrt{L_1 L_2}}$

- Generally $k<1$ always.

In case of strong coupling the difference of frequencies of two mode of vibration is more.
In case of loose coupling the system will behave as a single oscillator and vibrates with the natural frequency.