Ch -5

ELECTRIC POTENTIAL

Electric Potential

The electric potential *V* at a given point is the electric potential energy U of a small test charge q_0 situated at that point divided by the charge itself:

$$V = \frac{U}{q}.$$

If we set $U_i = 0$ at infinity as our reference potential energy,

$$\Delta V = V_f - V_i = -\frac{W}{q} \quad \text{(potential difference defined)}.$$

$$V = -\frac{W_\infty}{q} \quad \text{(potential defined)},$$

SI Unit of Electric Potential: joule/coulomb=volt (V)

Note:

Both the electric potential energy U and the electric potential *V* are scalars.

The electric potential energy U and the electric potential *V* are *not* the same. The electric potential energy is associated with a test charge, while electric potential is the property of the electric field and does not depend on the test charge.

The Electric Potential Difference

The *electric potential difference* between any two points *i* and *f* in an electric field.

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} .$$

$$\Delta V = V_f - V_i = -\frac{W}{q} \quad \text{(potential difference defined)} .$$

- It is equal to the difference in potential energy per unit charge between the two points.
- the negative work done by the electric field on a unite charge as that particle moves in from point i to point *f*.

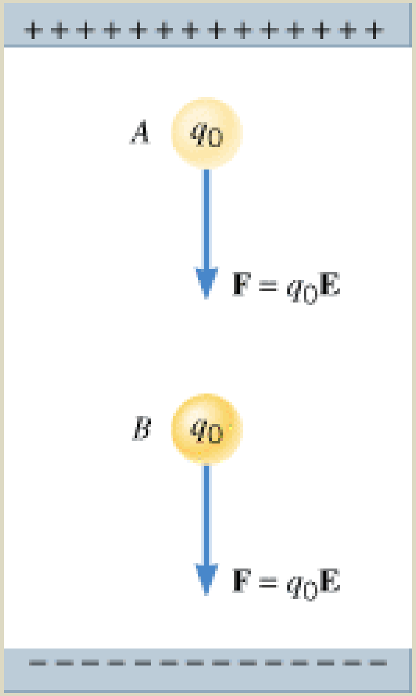
Note:

•Only the *differences ΔV* and *ΔU* are measurable in terms of the work *W.*

•The is *ΔV* property of the electric field and has nothing to do with a test charge

•The common name for electric potential difference is "voltage".

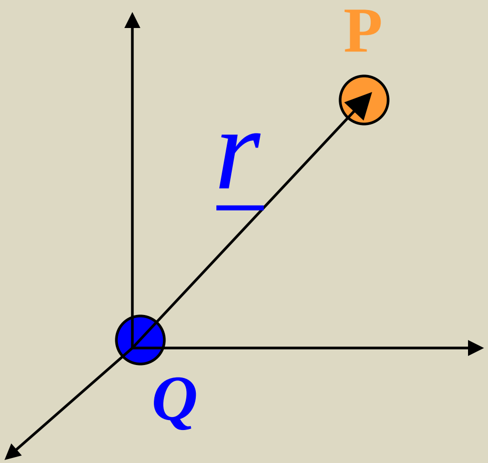
- **Electric field always points from higher electric potential to lower electric potential.**
- **A positive charge accelerates from a region of higher electric potential energy (or higher potential) toward a region of lower electric potential energy (or lower potential).**
- **A negative charge accelerates from a region of lower potential toward a region of higher potential.**



Electrostatic Potential of a Point Charge at the Origin

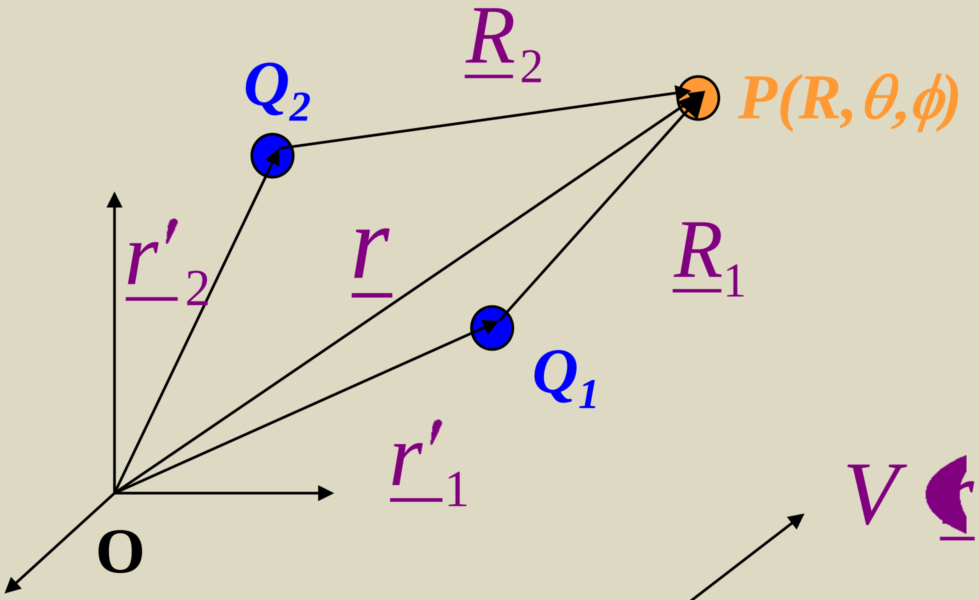
$$V(r) = -\int_{\infty}^{r} \overline{E} \cdot d\overline{l} = -\int_{\infty}^{r} \hat{a}_r \frac{Q}{4\pi\varepsilon_0 (r')^2} \cdot \hat{a}_r \, dr'$$

$$= \frac{Q}{4\pi\varepsilon_0} \int_{r}^{\infty} \frac{dr'}{(r')^2} = \frac{Q}{4\pi\varepsilon_0 r}$$



spherically symmetric

Electrostatic Potential Resulting from Multiple Point Charges



$$V(\underline{r}) = \sum_{k=1}^{n} \frac{Q_k}{4\pi\varepsilon_0 R_k}$$

No longer spherically symmetric!

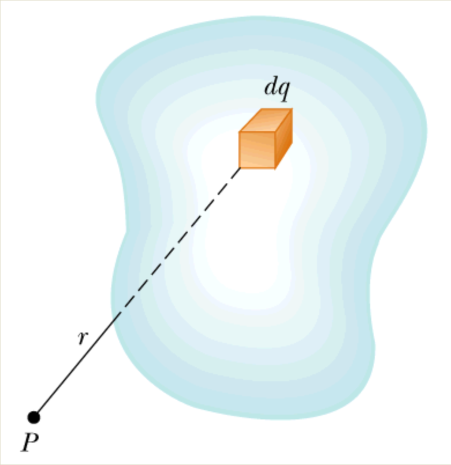
Potential Due to a Group of Point Charges

The potential at a point due to any number of point charges can be found by simply finding the potential at the point due to each alone and adding the potentials: $V_{tot}=V_1+V_2+\cdots+V_N$

$$V=\sum_{i=1}^{n} V_i=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \quad (n \text{ point charges}) .$$

Electric Potential due to continuous charge distribution

(1) Consider potential due to small charge element *dq*, treating this element as a point charge. The electric potential *dV* at some point *P* due to *dq* is



$$dV = k \frac{dq}{r}$$

Summing up all elements ➔

$$V = k \int \frac{dq}{r}$$

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Electric Potential due to continuous charge distribution

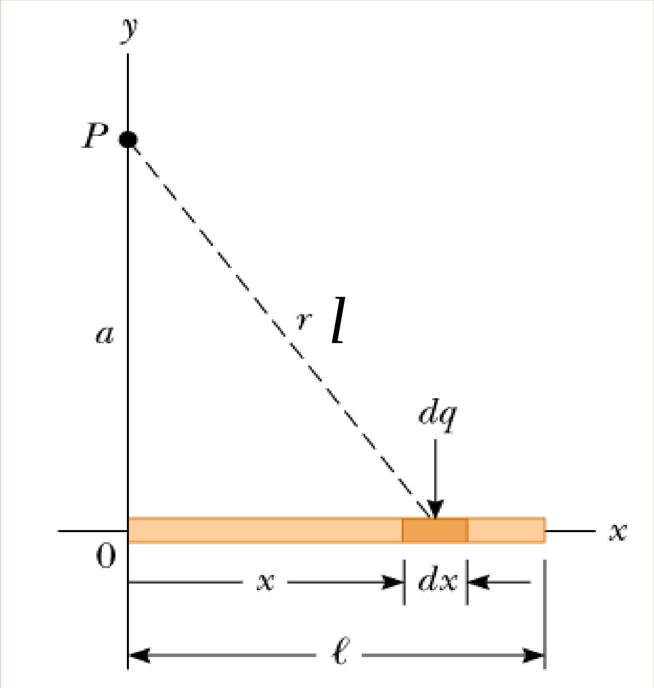
(2) If electric field is already known from other considerations (Gauss's Law), we can calculate the electric potential due to a continuous charge distribution using

$$\Delta V = - \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

First determine D*V* between any two points and then choose the electric potential V to be zero at some convenient point.

Electric Potential due to a finite line of charge

A rod of length *l* located along the x axis has a total charge Q and a uniform linear charge density l=Q/ *l* . Find the electric potential at a point P located on the y axis at distance *a* from the origin.



Length element *dx* with charge *dq* = *l dx*

$$dV = k \frac{dq}{r} = k \frac{\lambda dx}{\sqrt{x^2 + a^2}}$$

Integrate *dV* over limits x=0 to x = *l*

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Electric Potential due to a finite line of charge

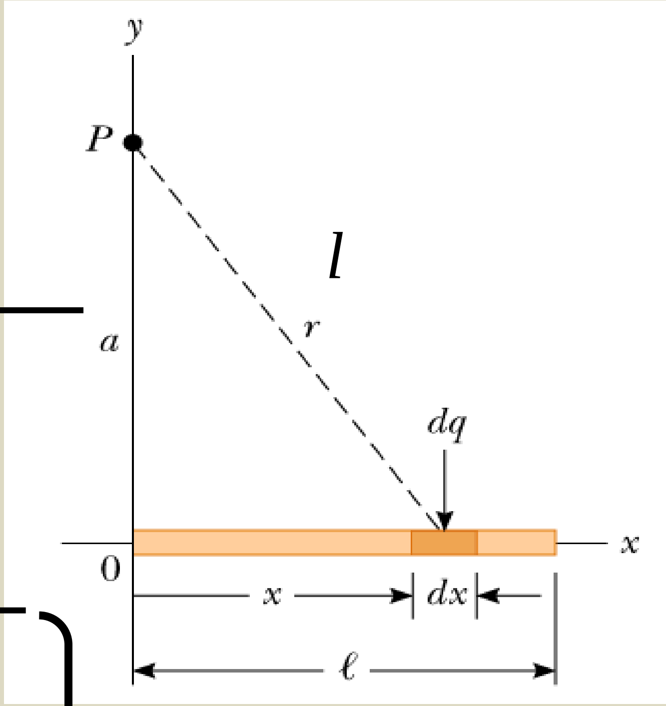
Integrate dV over limits x=0 to x = l

$$V = k\lambda \int_0^{l} \frac{dx}{\sqrt{x^2 + a^2}}$$

Note $\lambda = Q/l$ and

Natural Log

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right)$$



We have

$$V = \frac{kQ}{l} \ln\left(\frac{l + \sqrt{l^2+a^2}}{a}\right)$$

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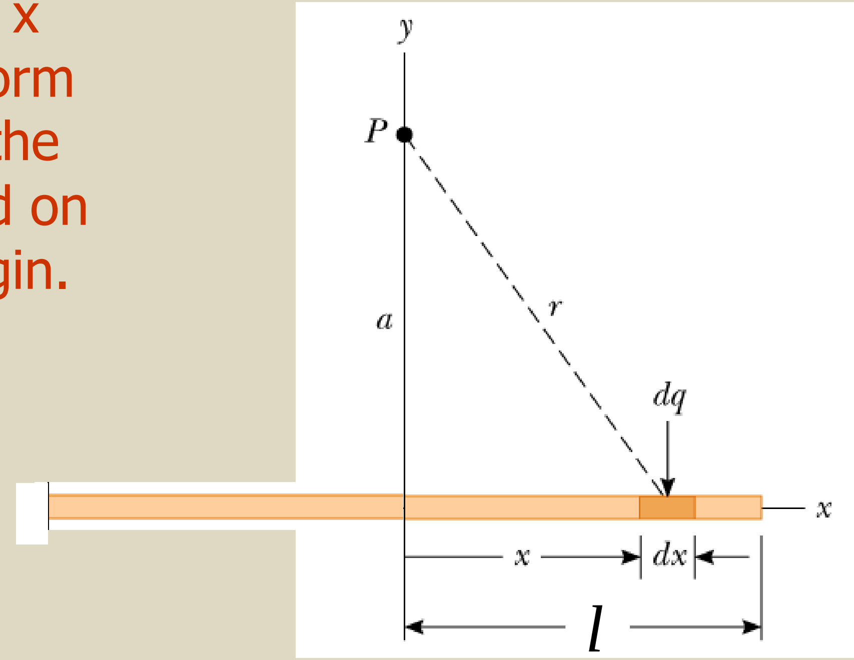
Electric Potential due to a finite line of charge

A rod of length 2 l located along the x axis has a total charge Q and a uniform linear charge density $\lambda = Q/2l$. Find the electric potential at a point ***P*** located on the y axis a distance *a* from the origin.



$$V = k\lambda \int_{-l}^{l} \frac{dx}{\sqrt{x^2 + a^2}}$$

$$V = \frac{kQ}{2l} \ln\left(\frac{\sqrt{l^2+a^2} + l}{\sqrt{l^2+a^2} - l}\right)$$

How is this result consistent with the E field for infinite line of charge obtained using Gauss's Law? (Homework)

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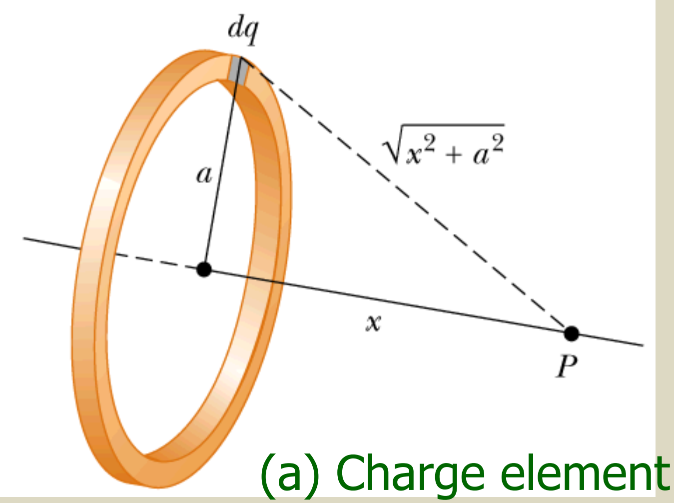
Electric Potential due to a uniformly charged ring



(a) Find an expression for the electric potential at point ***P*** located on the perpendicular central axis of a uniformly charged ring of radius *a* and total charge Q. (b) Find an expression for the magnitude of the electric field at point P.

(a) Charge element *dq* is at a distance $\sqrt{x^2+a^2}$ from point *P*.

$$V = k\int \frac{dq}{r} = k\int \frac{dq}{\sqrt{x^2 + a^2}}$$

And each element dq is at the same distance from P, i.e.

$$V = \frac{k}{\sqrt{x^2 + a^2}}\int dq = \frac{kQ}{\sqrt{x^2 + a^2}}$$

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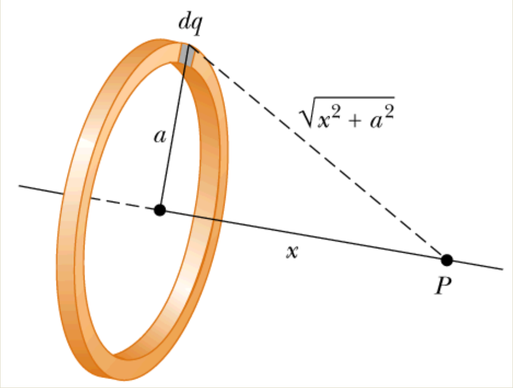
Electric Potential due to a uniformly charged ring

(b) Use $E_x = - dV/dx$

$$E_x = - \frac{dV}{dx} = - kQ \frac{d}{dx} (x^2 + a^2)^{-1/2}$$



$$= - kQ \left(- \tfrac{1}{2}\right) (x^2 + a^2)^{-3/2} \, 2x$$

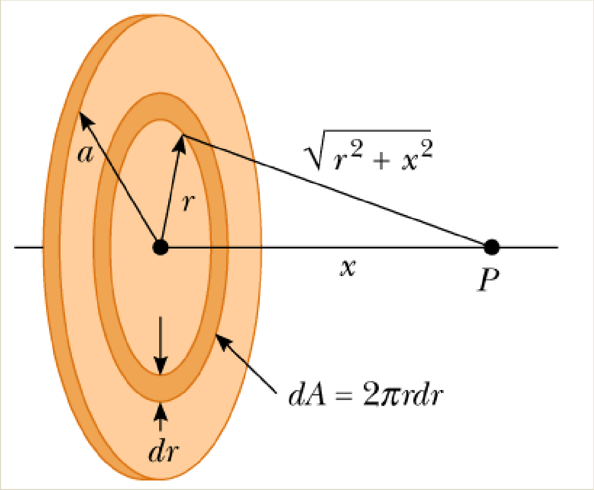
$$E_x = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

What about E_y and E_z ?

What is the electric potential at the center of the ring?

What is the electric field at the center of the ring?

Electric Potential due to a uniformly charged disk



(a) Find an expression for the electric potential at point ***P*** located on the perpendicular central axis of a uniformly charged disk of radius a and surface charge density σ. (b) Find an expression for the magnitude of the electric field at point P.

(a) Divide into rings radius *r* and width *dr* and surface area *dA=2πr dr*

$$dV = \frac{k\,dq}{\sqrt{x^2 + r^2}} = \frac{k\sigma 2\pi r dr}{\sqrt{x^2 + r^2}}$$

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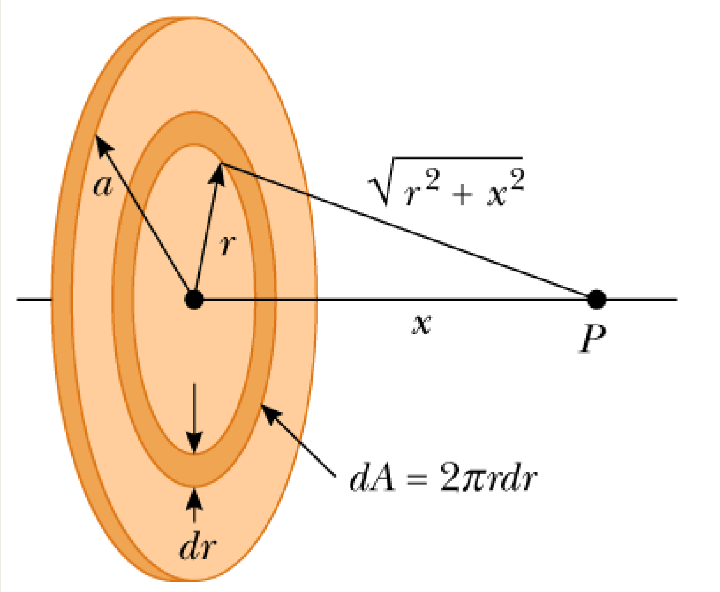
Electric Potential due to a uniformly charged disk

(a) To find the potential, sum over all rings.
Integrate *dV* from r= 0 to r= a:

$$V = \pi k\sigma \int_0^a \frac{2r\,dr}{\sqrt{x^2 + r^2}}$$

$$V = 2\pi k\sigma \left[(x^2 + a^2)^{1/2} - x \right]$$



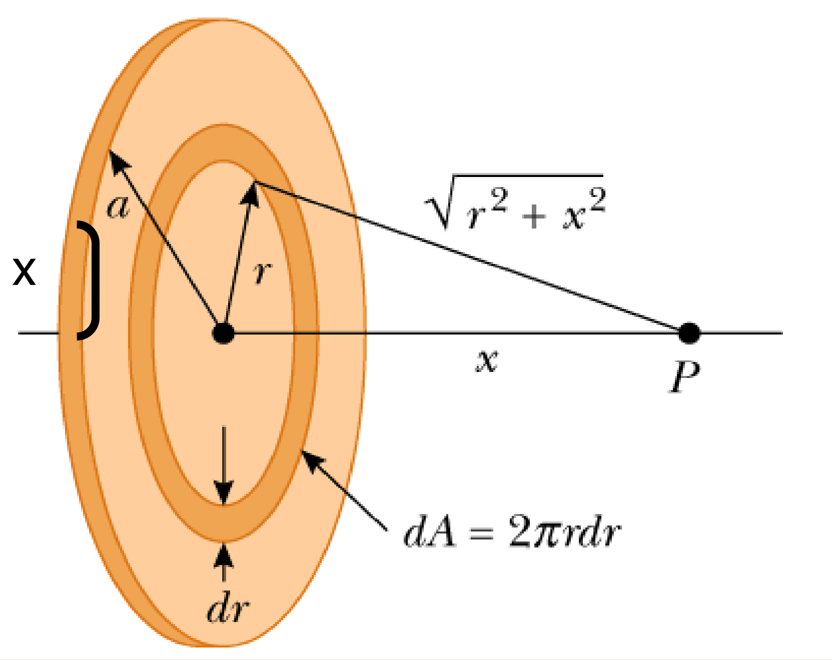
Electric Potential due to a uniformly charged disk

(b) $E_x = - dV/dx$

$$E_x = -\frac{dV}{dx}$$

$$= -\frac{d}{dx} 2\pi k\sigma \left[(x^2 + a^2)^{1/2} - x \right]$$

$$E_x = 2\pi k\sigma \left(1 - \frac{x}{\sqrt{x^2 + a^2}} \right)$$



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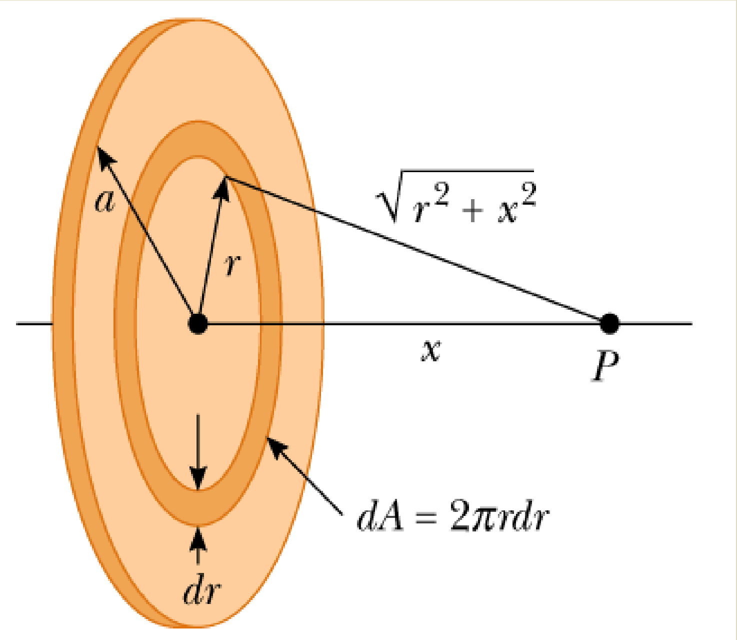
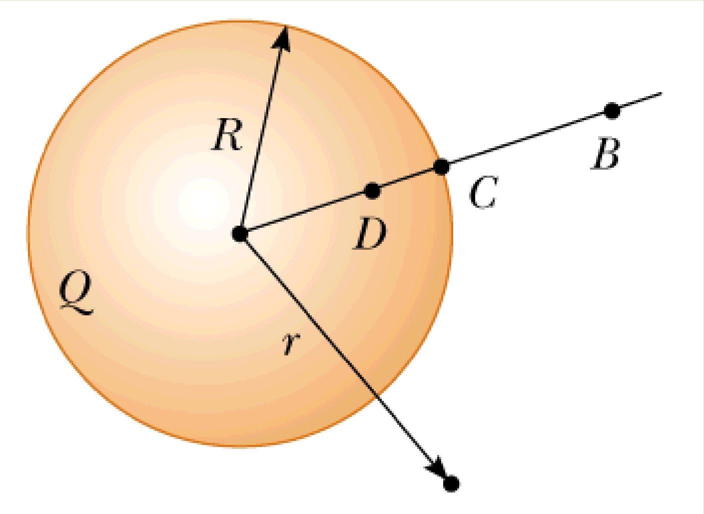
Electric Potential due to a uniformly charged disk

$$E_x = 2\pi k\sigma \left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$$



When you are really close to this disk, then it is as if you are looking at an infinite plane of charge, use above equation to deduce the electric field. Is the result consistent with the result obtained from our discussion using Gauss's law ?

Electric Potential due to a uniformly charged sphere



An insulating solid sphere of radius R has a uniform positive volume charge density and total charge Q.

(a) Find the electric potential at a point outside the sphere, that is, $r > R$. Take the potential to be zero at $r = \infty$.

(b) Find the potential of a point inside the sphere, ($r < R$).

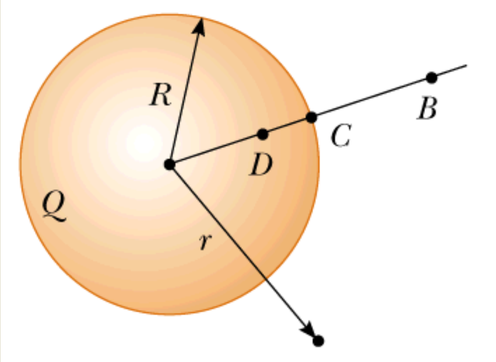
In this case, it is easier to use electric field obtained in our previous discussions and determine the electric potential.

Electric Potential due to a uniformly charged sphere

Outside the sphere, we have

$$E_r = \frac{k\,Q}{r^2} \qquad \text{For } r > R$$

To obtain potential at B, we use



$$V_B = - \int^{r} E_r\,dr = - kQ \int_{\infty}^{r} \frac{dr}{r^2}$$

$$V_B = \frac{k\,Q}{r}$$

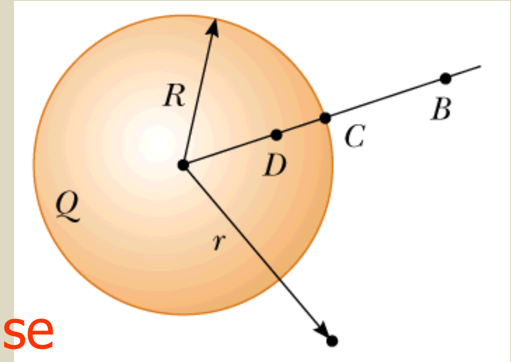
Potential must be continuous at r = R, => potential at surface

$$V_C = \frac{k\,Q}{R}$$

Pictures from Serway & Beichner

Electric Potential due to a uniformly charged sphere

Inside the sphere, we have



$$E_r = \frac{k\,Q}{R^3}\, r \qquad \text{For } r < R$$

To obtain the potential difference at D, we use

$$V_D - V_C = -\int_R^r E_r\, dr = -\frac{k\,Q}{R^3}\int_R^r r\, dr = \frac{k\,Q}{2R^3}\left(R^2 - r^2\right)$$

Since $$V_C = \frac{k\,Q}{R}$$

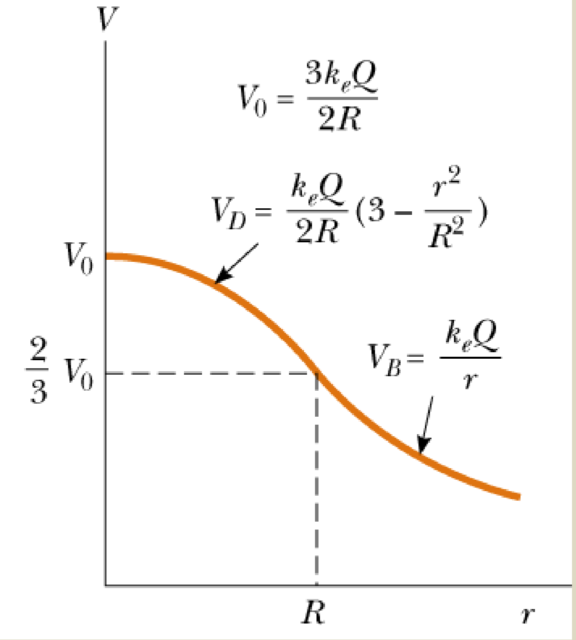
To obtain the absolute value of the potential at D, we add the potential at C to the potential difference $V_D - V_C$:

$$V_D = \frac{k\,Q}{2R}\left(3 - \frac{r^2}{R^2}\right) \qquad \text{For } r < R$$

Check V for r = R

Pictures from Serway & Beichner

Electric Potential due to a uniformly charged sphere



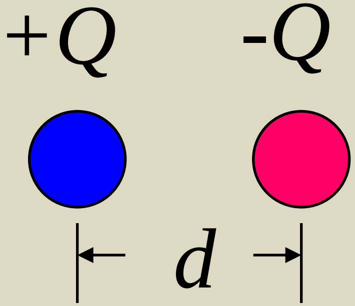
What are the magnitude of the electric field and the electric potential at the center of the sphere?

A plot of electric potential V versus distance r from the center of a uniformly charged insulating spheres of radius R. The curve for V_D inside the sphere is parabolic and joined smoothly with the curve for V_B outside the sphere, which is a hyperbola. The potential has a maximum value Vo at the center of the sphere.

What are the differences if the sphere is a conducting sphere?

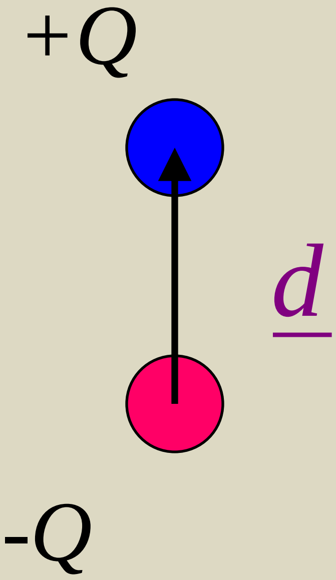
Charge Dipole

- An ***electric charge dipole*** consists of a pair of equal and opposite point charges separated by a small distance (i.e., much smaller than the distance at which we observe the resulting field).



Dipole Moment

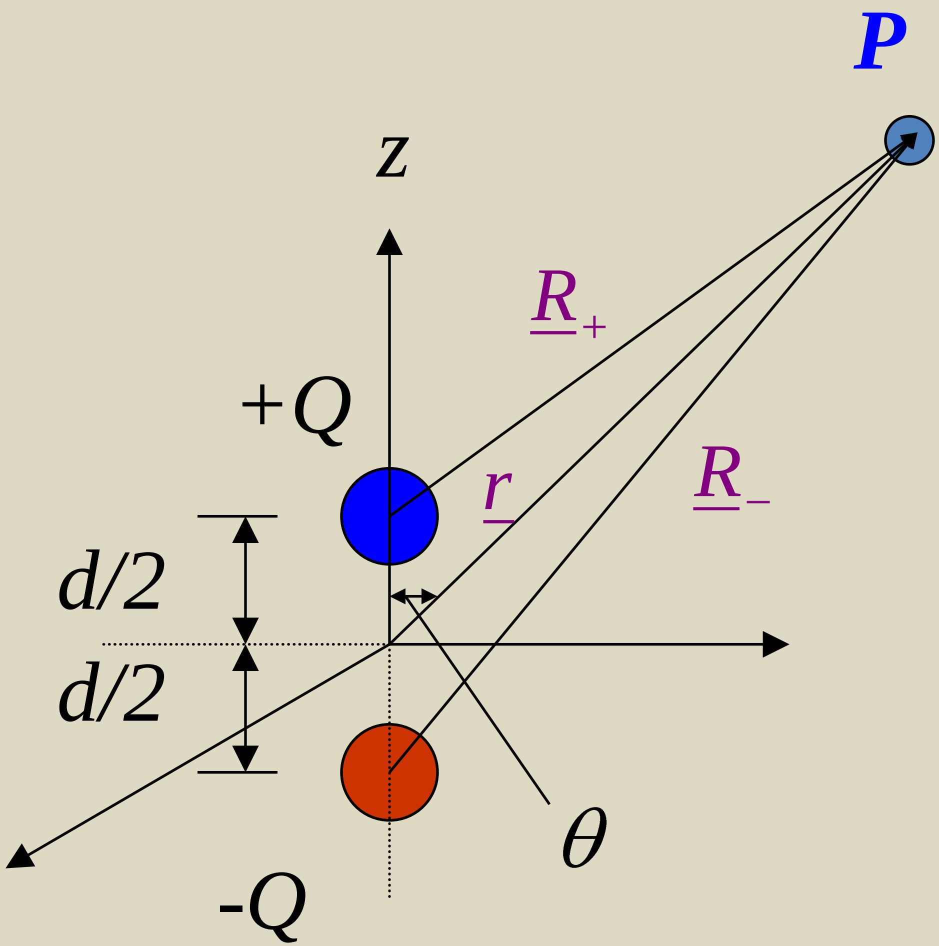
- ***Dipole moment p*** is a measure of the strength of the dipole and indicates its direction



$$\underline{p} = Q\underline{d}$$

p *is in the direction from the negative point charge to the positive point charge*

Electrostatic Potential Due to Charge Dipole



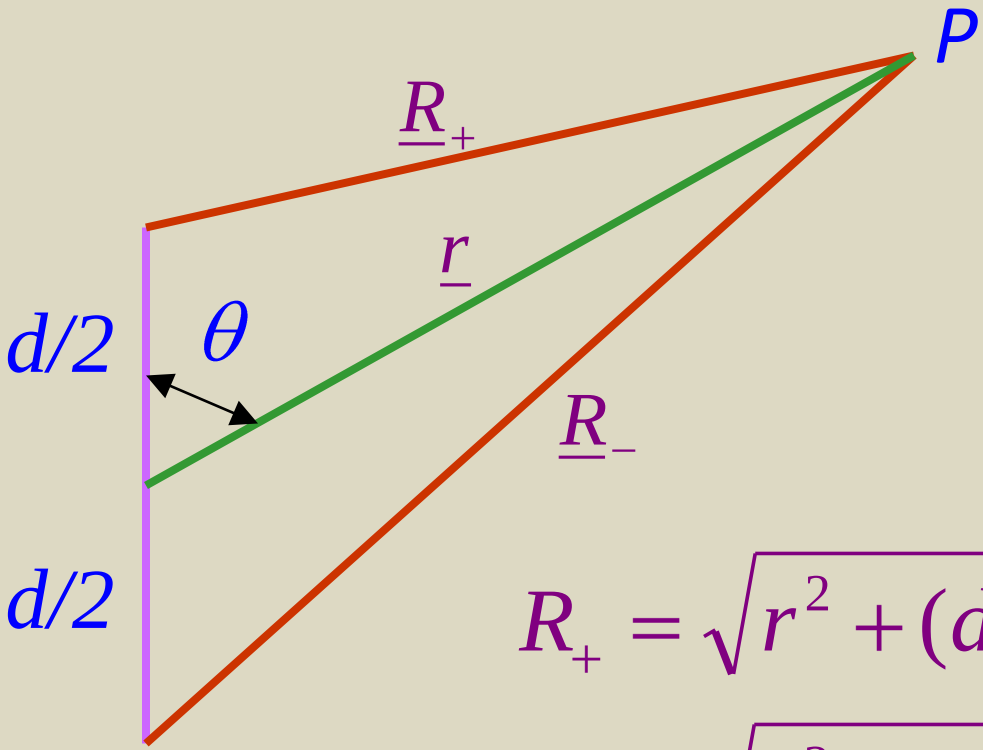
$$\underline{p} = \hat{a}_z Q d$$

Electrostatic Potential Due to Charge Dipole (Cont'd)

$$V(\underline{r}) = V(r,\theta) = \frac{Q}{4\pi\varepsilon_0 R_+} - \frac{Q}{4\pi\varepsilon_0 R_-}$$

cylindrical symmetry

Electrostatic Potential Due to Charge Dipole (Cont'd)



$$R_{+} = \sqrt{r^{2} + (d/2)^{2} - rd\cos\theta}$$

$$R_{-} = \sqrt{r^{2} + (d/2)^{2} + rd\cos\theta}$$

Electrostatic Potential Due to Charge Dipole in the Far-Field

- assume $R >> d$

- *zeroth order* approximation:

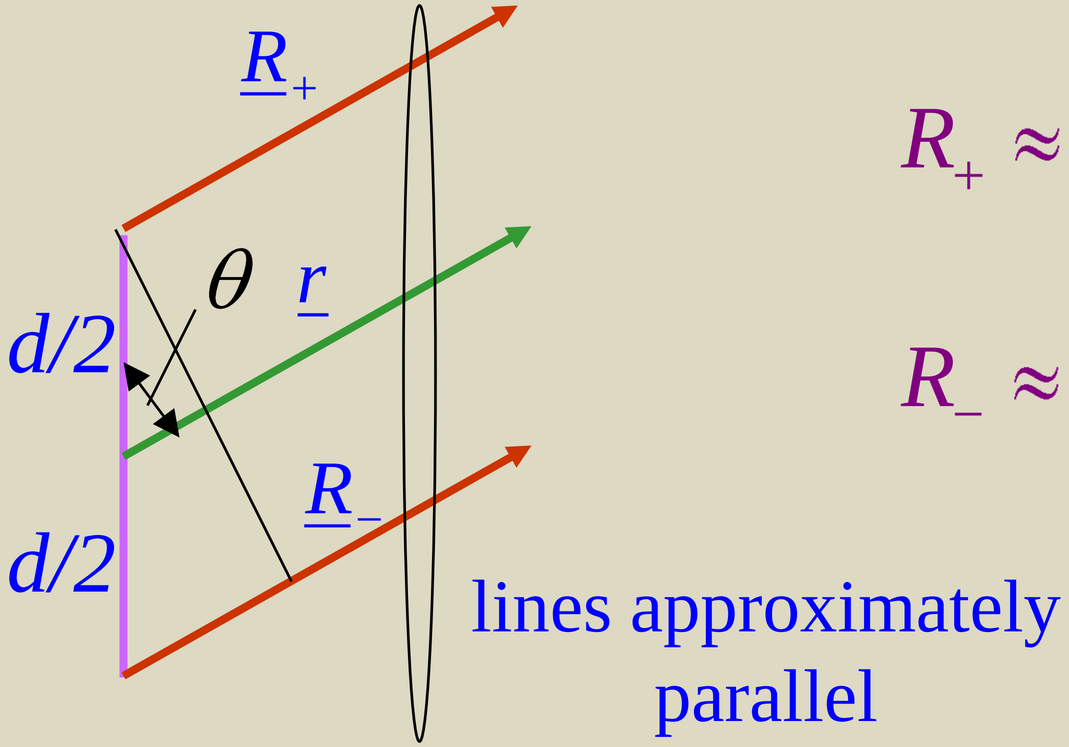
$$R_+ \approx R$$
$$R_- \approx R$$

$$\rightarrow \quad V \approx 0$$

not good enough!

Electrostatic Potential Due to Charge Dipole in the Far-Field (Cont'd)

- *first order* approximation from geometry:



$$R_+ \approx r - \frac{d}{2}\cos\theta$$

$$R_- \approx r + \frac{d}{2}\cos\theta$$

Electrostatic Potential Due to Charge Dipole in the Far-Field (Cont'd)

- *Taylor series* approximation:

$$\frac{1}{R_+} = \left\{ r - \frac{d}{2}\cos\theta \right\}^{-1} = \frac{1}{r}\left\{ 1 - \frac{d}{2r}\cos\theta \right\}^{-1}$$

$$\approx \frac{1}{r}\left(1 + \frac{d}{2r}\cos\theta \right)$$

$$\frac{1}{R_-} \approx \frac{1}{r}\left(1 - \frac{d}{2r}\cos\theta \right)$$

Recall :

$$(1+x)^n \approx 1 + nx, \quad x << 1$$

Electrostatic Potential Due to Charge Dipole in the Far-Field (Cont'd)

$$V(r,\theta) \approx \frac{Q}{4\pi\varepsilon_0 r}\left[\left(1+\frac{d\cos\theta}{2r}\right)-\left(1-\frac{d\cos\theta}{2r}\right)\right]$$

$$= \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

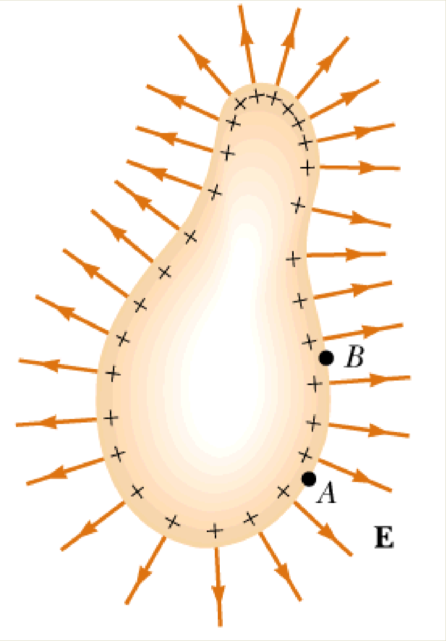
Electrostatic Potential Due to Charge Dipole in the Far-Field (Cont'd)

- In terms of the *dipole moment*:

$$V \approx \frac{1}{4\pi\varepsilon_0}\frac{\underline{p}\cdot\hat{a}_r}{r^2}$$

Electric Potential due to a charged conductor

Every point on the surface of a charged conductor in equilibrium is at the same electric potential.



Consider two points A and B on the surface of a charged conductor as shown. Along a surface path connecting these points, E is always perpendicular to the displacement ds; therefore E.ds = 0.

Using this result and

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = \mathbf{0}$$

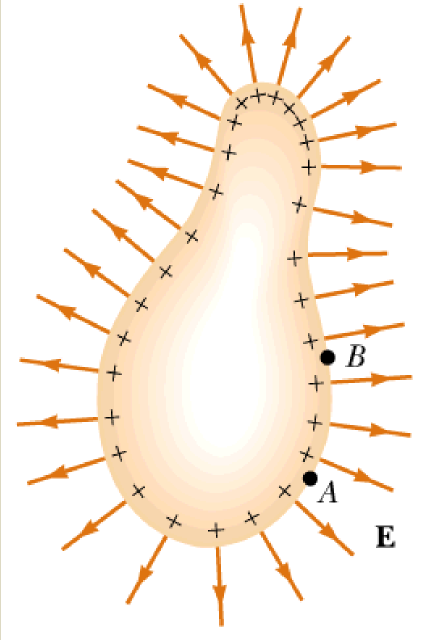
We find potential difference between A and B is = zero.
This result applies to any two points on the surface

Pictures from Serway & Beichner

Electric Potential due to a charged conductor

What about inside the conductor?

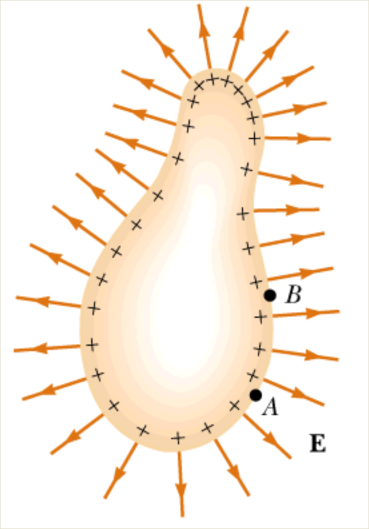
Because the electric field is zero inside the conductor, we conclude from the relationship $E = -dV/dr$ that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.



What is the work done in moving a positive charge from the interior of a charged conductor to its surface?

Pictures from Serway & Beichner

Electric Potential due to a charged conductor



An arbitrarily shaped conductor carrying a positive charge. When the conductor is in electrostatic equilibrium, all of the charge resides at the surface, E = 0 inside the conductor, and the direction of E just outside the conductor is perpendicular to the surface. The electric potential is constant inside the conductor and is equal to the potential at the surface.

Note from the spacing of the plus (+) signs that the surface charge density is non-uniform.

Surface charge density is high where the radius of curvature is small and the surface is convex. And vice-versa.

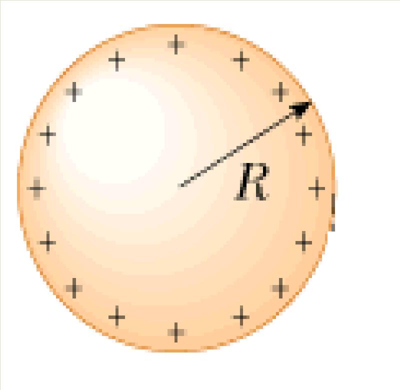
Because E field just outside the conductor is proportional to the surface charge density, we see that the **electric field is large near convex points having small radii of curvature and reaches very high values at sharp points.**

Electric Potential due to Spherical Charged Conductor

Outside the sphere, we have

$$E_r = \frac{k\,Q}{r^2} \qquad \text{For } r > R$$

To obtain potential at B, we use

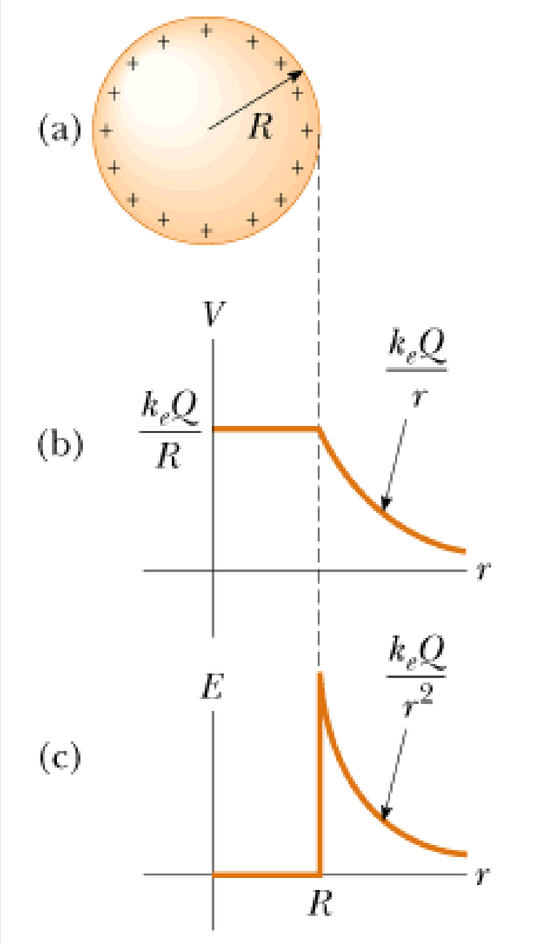


$$V_B = -\int^{r} E_r\,dr = -\,kQ\int_{\infty}^{r} \frac{dr}{r^2}$$

$$V_B = \frac{k\,Q}{r}$$

Pictures from Serway & Beichner

Electric Potential due to a charged conductor

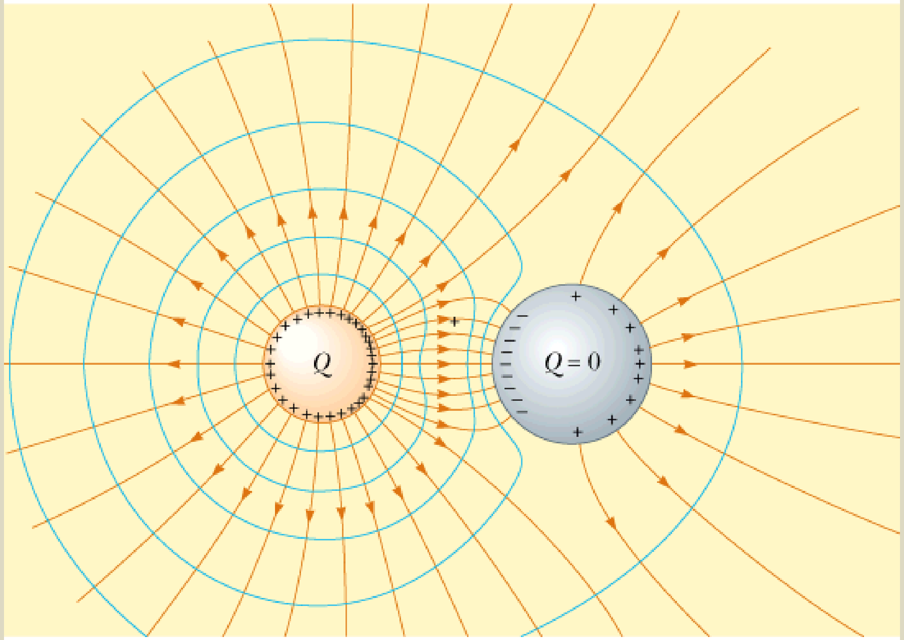


(a) The excess charge on a conducting sphere of radius R is uniformly distributed on its surface.

(b) Electric potential versus distance r from the center of the charged conducting sphere.

(c) Electric field magnitude versus distance r from the center of the charged conducting sphere.

Electric Potential due to a charged conductor



The electric field lines (in orange) around two spherical conductors. The smaller sphere has a net charge Q, and the larger one has zero net charge. The blue curves are cross sections of equipotential surfaces.

Note that the surface charge density is not uniform

Pictures from Serway & Beichner

Electric Field from Electric Potential

Electric field related to electric potential by

$$\Delta V = \Delta U / q_o = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

This means

$$dV = -\mathbf{E} \cdot d\mathbf{s}$$

If electric field has only one component E_x, then E . ds = E_x dx. We have dV = – E_x dx or

$$E_x = -\frac{dV}{dx}$$

Electric Field from Electric Potential

$$E_x = -\frac{dV}{dx}$$

- Magnitude of E field in the direction of some coordinate is equal to negative of the derivative of the electric potential w.r.t. that coordinate.

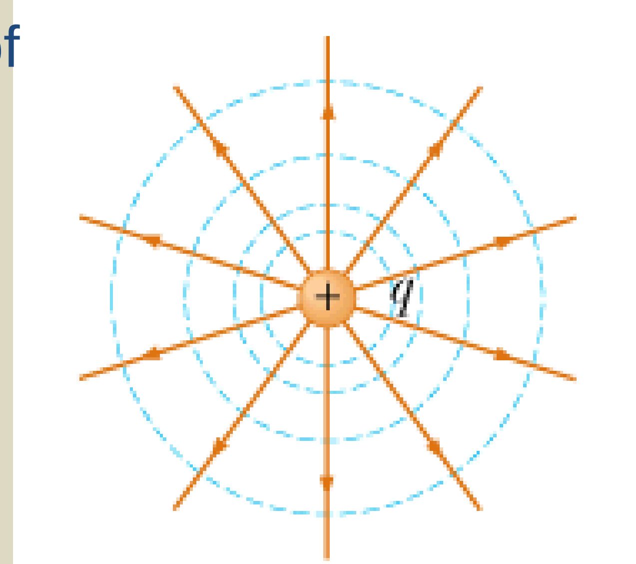
- If the charge distribution creating the E-field has spherical symmetry such that the volume charge density depends only on the radial distance r, then the electric field is radial. In this case, and $\mathbf{E} \cdot d\mathbf{s} = E_r \, dr$

$$E_r = -\frac{dV}{dr}$$

Electric Field from Electric Potential

$$E_r = -\frac{dV}{dr}$$

- For point charge $V = kq/r$ and we have $E = kq/r^2$
- The potential changes only in the radial direction, not in any direction perpendicular to r. Thus V is a function only of r.
- Equipotential surfaces are a family of spheres concentric with the spherically symmetric charge distribution.
- Equipotential surface perpendicular to field lines.



Pictures from Serway & Beichner

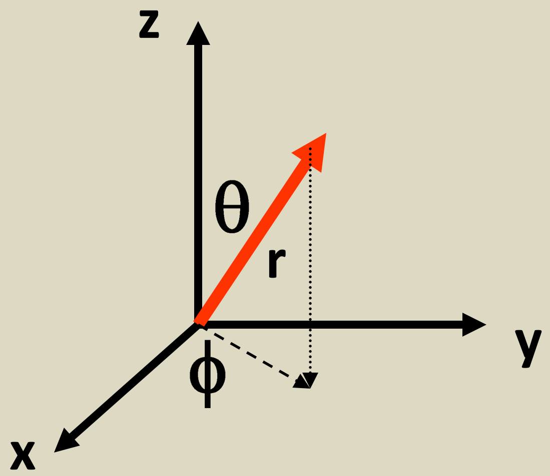
Electric Field from Electric Potential

- More general expression (in cartesian coordinate)

$$\mathbf{E} = -\left(\frac{dV}{dx}\hat{x} + \frac{dV}{dy}\hat{y} + \frac{dV}{dz}\hat{z}\right)$$

- More general expression (in spherical coordinate)

$$\mathbf{E} = -\left(\frac{dV}{dr}\hat{r} + \frac{1}{r}\frac{dV}{d\theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{dV}{d\phi}\hat{\phi}\right)$$



From general expression to simplified expression

- More general expression (in spherical coordinate)

$$\mathbf{E} = -\left(\frac{dV}{dr}\hat{r} + \frac{1}{r}\frac{dV}{d\theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{dV}{d\phi}\hat{\phi}\right)$$

If the potential V does no depends on the coordinates θ and ϕ, then $dV/d\theta = 0$ and $dV/d\phi = 0$, we have

$$\mathbf{E} = -\frac{dV}{dr}\hat{r}$$

Poisson's Equation

$$\nabla^2 V = -\frac{q_{ev}}{\varepsilon_0}$$

Poisson's equation

$\Rightarrow$ ∇^2 is the *Laplacian operator*. The *Laplacian* of a scalar function is a scalar function equal to the divergence of the gradient of the original scalar function.

Laplace's Equation

- ***Laplace's equation*** is the homogeneous form of ***Poisson's equation***.
- We use Laplace's equation to solve problems where potentials are specified on conducting bodies, but no charge exists in the free space region.

$$\nabla^2 V = 0$$

Laplace's equation

Electric Potential Energy

- Test charge q_o placed in an electric field **E** created by some other charged object. We get electric force q_o**E**.
- Coulomb's force is a conservative force.
- External agent displaces the charge, then work done by the field is equal to work done by external agent causing the displacement.
- **(REMEMBER: Work = force x displacement)**
- For an infinitesimal displacement d**s**, the work done, dW, by the electric field on the charge is **F . ds =** q_o**E . ds**.

Electric Potential Energy

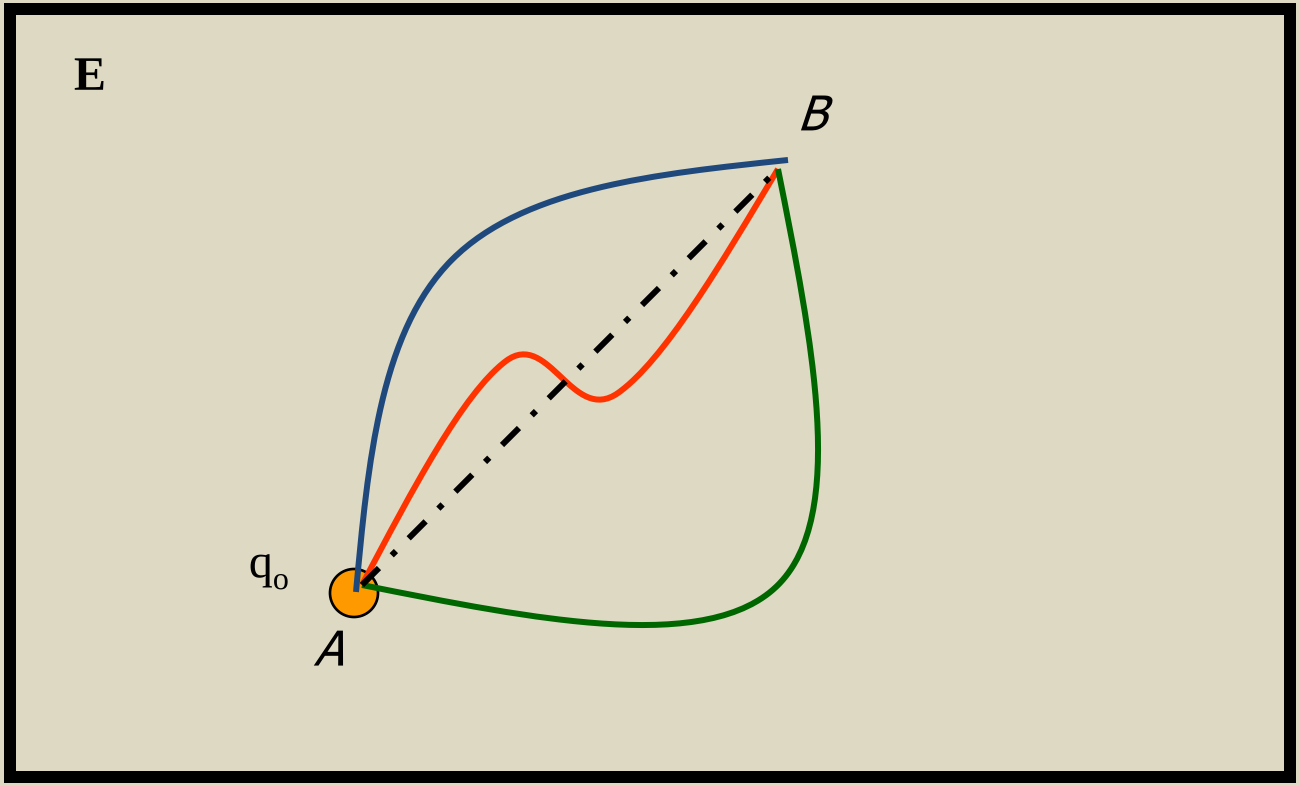
- The potential energy of the charge-field system is decreased by an amount $dU = -q_o\mathbf{E} \cdot d\mathbf{s} = -dW$
- For a finite displacement of the charge from a point *A* to a point *B*, the change in potential energy of the system $\Delta U = U_B - U_A$ is

$$\Delta U = -q_o \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Integration performed along the path q_o follows as it moves from *A* to *B* (called path integral or line integral).
Force is conservative, this line integral does not depend on the path taken from *A* to *B*.

Conservative Force

$$\Delta U = -q_o \int_A^B \mathbf{E} \cdot d\mathbf{s}$$



If the path between *A* to *B* does not make any difference, why don't we just use the expression $\Delta U = -q_o E d$ where d is the straight-line distance between *A* and *B*?

Electric Potential

• The Potential Energy per unit charge U/q_o is independent of the value of q_o and has a unique value at every point in an electric field. This quantity is called the electric potential (or simply the potential) *V.*

$$V = U/q_o$$

• Potential difference $\Delta V = V_B - V_A$ between any two points A and B is

$$\Delta V = \Delta U / q_o = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Electric Potential at an arbitrary point

- Electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity (where V =0) to that point.

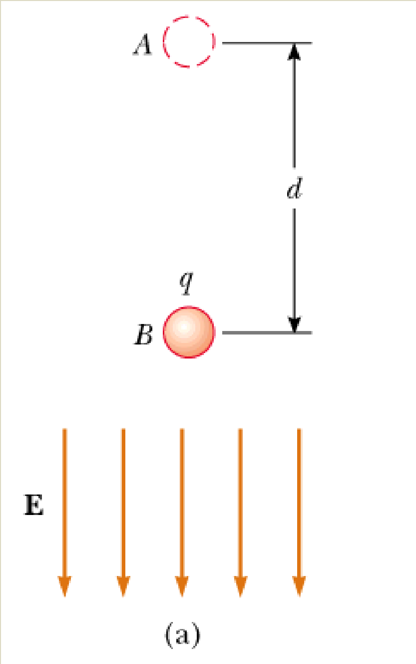
- Electric potential at any point *P* is

$$V_p = -\int_{\infty}^{P} \mathbf{E} \cdot d\mathbf{s}$$

S.I. unit J/C defined as a volt (V) and 1 V/m = 1 N/C

Note that V_p represents the potential difference ΔV between the point P and a point at infinity.

Potential Differences in Uniform E field



Example: Uniform field along –y axis (E parallel to ds)

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E\, ds$$

$$\Delta V = -E\int_A^B ds = -E\,d$$

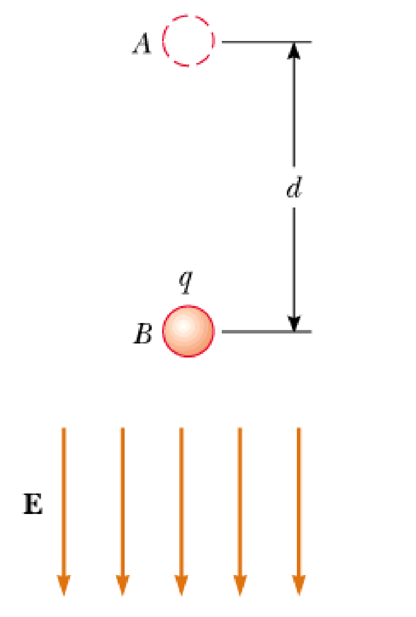
- When the electric field E is directed downward, point B is at a lower electric potential than point A. A positive test charge that moves from point A to point B loses electric potential energy.
- **Electric field lines *always* point in the direction of decreasing electric potential.**

Potential Energy in Uniform E field

Example: Uniform field along –y axis (E parallel to ds)

And suppose a test charge q_o moves from A to B.

We have $\Delta U = q_o \Delta V = -q_o E d$



(a)

If q_o is positive then D*U* is negative. i.e. a positive charge loses electric potential energy when it moves in the direction of the electric field.

This means electric field does work on a positive charge when the charge moves in the direction of the electric field.

Release the test charge at rest, it will accelerate downward, gaining kinetic energy.

As the charged particle gains kinetic energy, it loses an equal amount of potential energy.

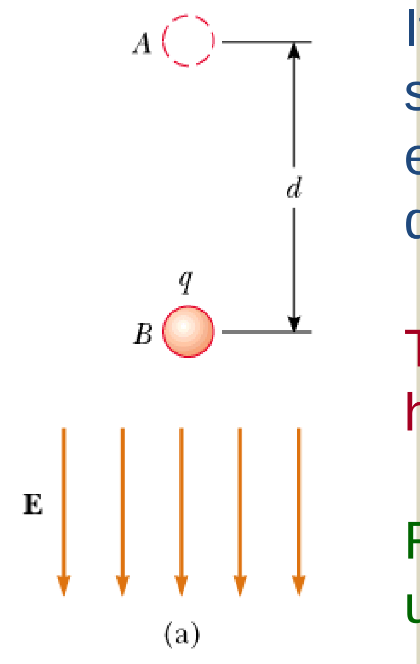
Pictures from Serway & Beichner

Potential Energy in Uniform E field

Example: Uniform field along –y axis (E parallel to ds)

And suppose a test charge q_o moves from A to B.

We have $\Delta U = q_o \Delta V = -q_o E d$



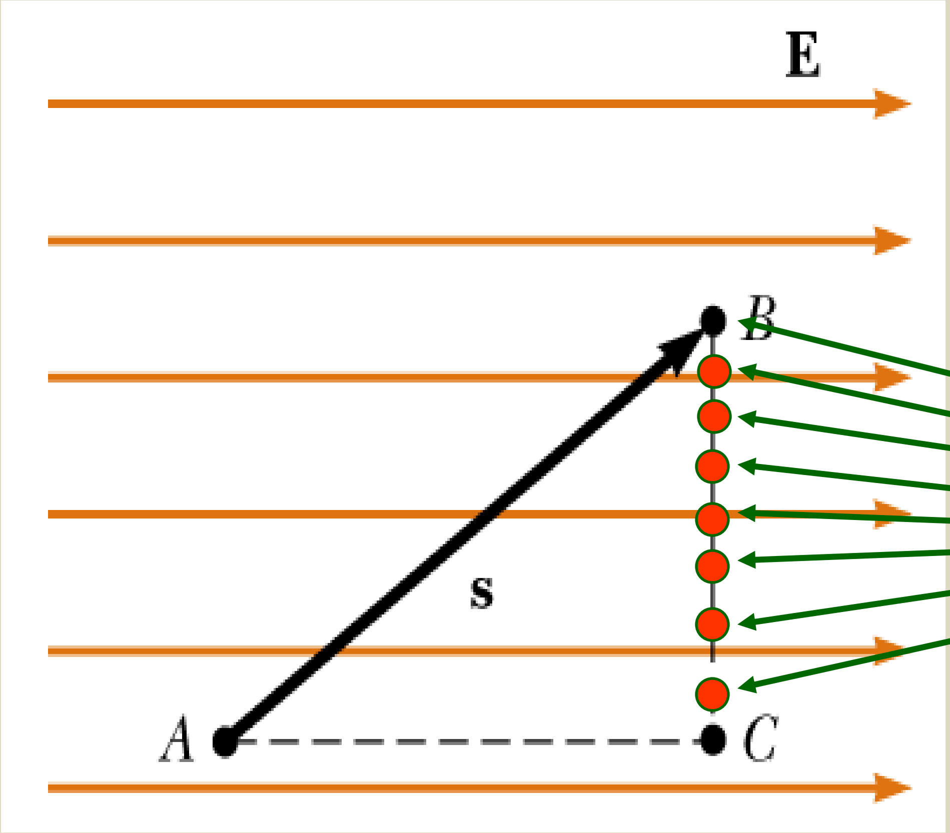
If q_o is negative then DU is positive and the situation is reversed: a negative charge gains electric potential energy when it moves in the direction of the electric field.

The external agent has to do work to cause this to happen.

Release the test charge at rest, it will accelerate upward in the direction opposite to electric field.

Pictures from Serway & Beichner

Equipotential



$$\Delta V = -\int_{C}^{B} \mathbf{E} \cdot d\mathbf{s} = 0$$

$V_C = V_B$ *(same potential)*

In fact, points along this line has the same potential. We have an equipotential line.

Pictures from Serway & Beichner

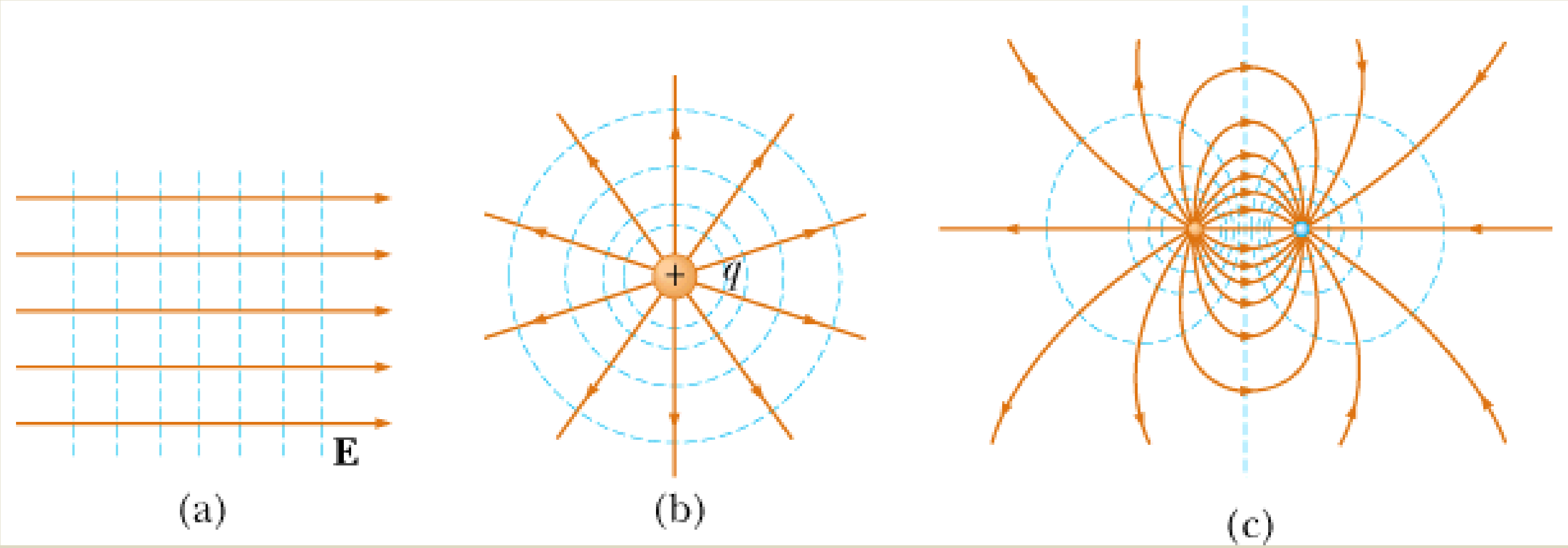
Equipotential Surfaces

The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.

No work is done in moving a test charge between any two points on an equipotential surface.

The equipotential surfaces of a uniform electric field consist of a family of planes that are all perpendicular to the field.

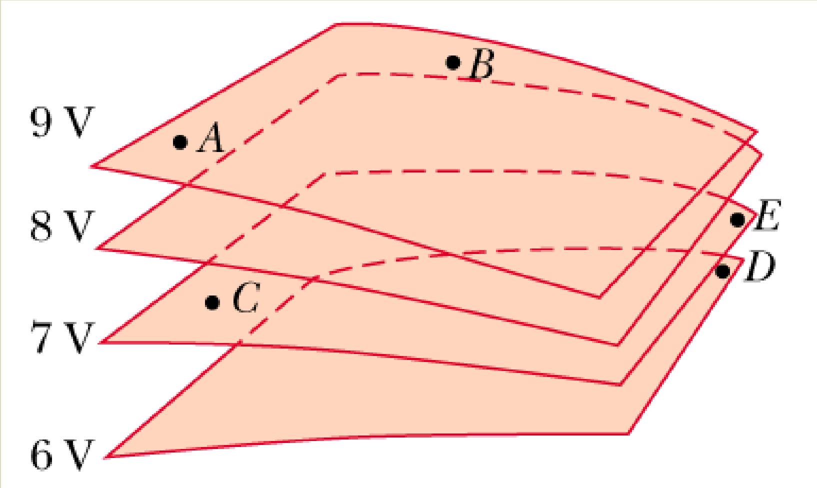
Equipotential Surface



Equipotential Surfaces (dashed blue lines) and electric field lines (orange lines) for (a) a uniform electric field produced by infinite sheet of charge, (b) a point charge, and (c) an electric dipole. In all cases, the equipotential surfaces are perpendicular to the electric field lines at every point.

Pictures from Serway & Beichner

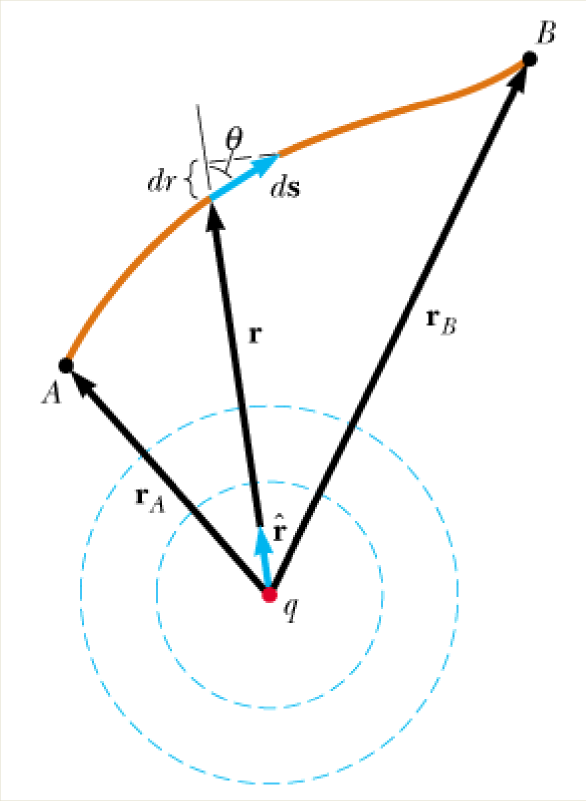
Equipotential Surfaces



Rank the work done by the E field on a positively charged particle that moves from (i) A to B; (ii) B to C; (iii) C to D; (iv) D to E.

Pictures from Serway & Beichner

Electric Potential and Potential Energy due to point charges



Consider isolated positive point charge q. (i.e. E directed radially outward from the charge)

To find electric potential at a point located at a distance r from the charge, start with the general expression for potential difference:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Where A and B are two arbitrary points as shown.

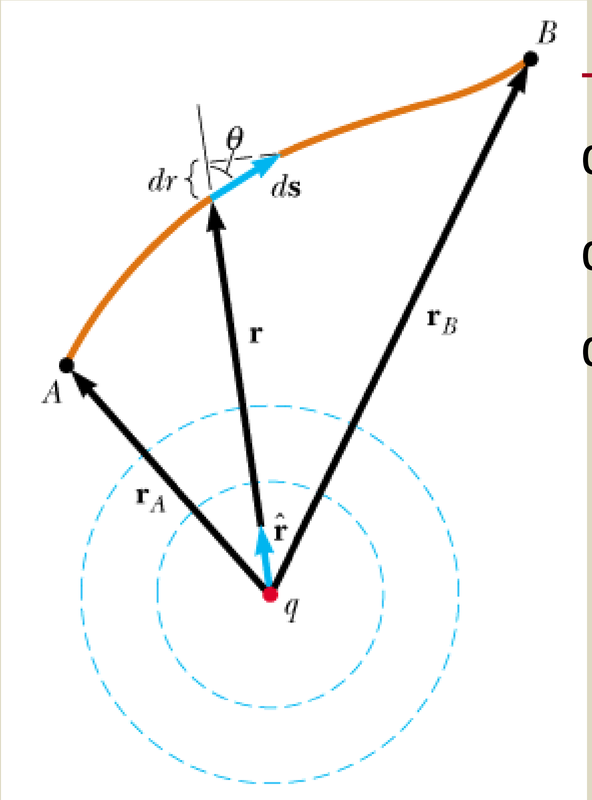
$\mathbf{E} = kq/r^2\,\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is a unit vector directed from the charge toward the field point.

Pictures from Serway & Beichner

Electric Potential and Potential Energy due to point charges

We can express $\mathbf{E} \cdot d\mathbf{s}$ as

$$\mathbf{E} \cdot d\mathbf{s} = kq/r^2 \; \hat{\mathbf{r}} \cdot d\mathbf{s}$$



The magnitude of $\hat{\mathbf{r}}$ is 1, dot product $\hat{\mathbf{r}} \cdot d\mathbf{s}$ = ds cos q, where q is the angle between $\hat{\mathbf{r}}$ and $d\mathbf{s}$.

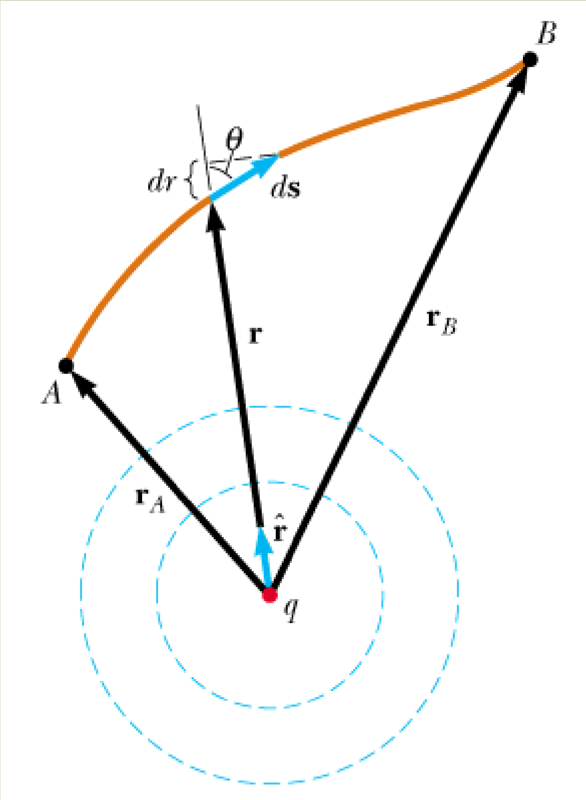
ds cos q is the projection of ds onto r , thus

ds cos q = dr.

$$V_B - V_A = - \int_A^B \mathbf{E} \cdot d\mathbf{s} = - \int_A^B kq/r^2 \, dr$$

Pictures from Serway & Beichner

Electric Potential and Potential Energy due to point charges



$$V_B - V_A = -\int_A^B E_r \, dr = -\int_{r_A}^{r_B} kq/r^2 \, dr$$

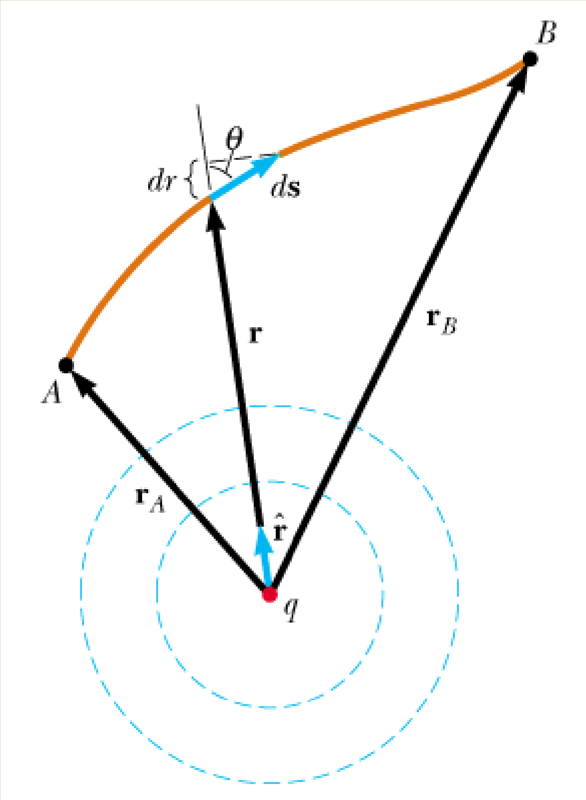
$$V_B - V_A = \left[\frac{kq}{r} \right]_{r_A}^{r_B}$$

$$V_B - V_A = kq \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

Depends only on the coordinates and not on the path.

Pictures from Serway & Beichner

Electric Potential and Potential Energy due to point charges



r_A = infinity (and $V_A = 0$), we have electric potential created by a point charge at a distance r from the charge given by

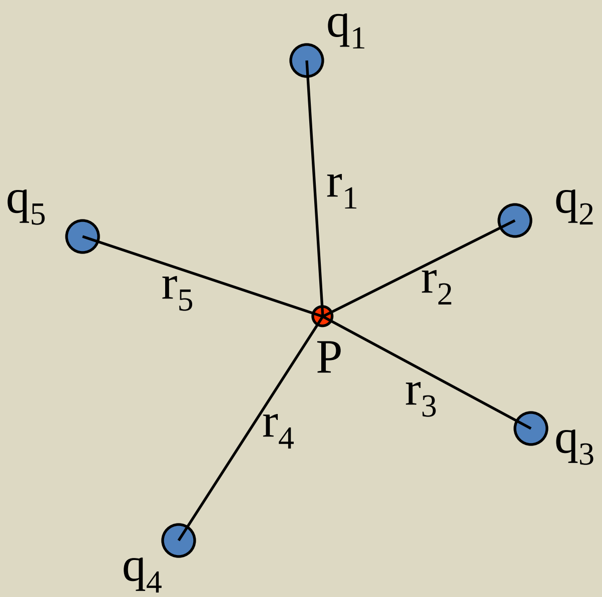
$$V = \frac{kq}{r}$$

Points at same distance r from q have the same potential V, i.e. the equipotential surfaces are spherical and centered on the charge.

Pictures from Serway & Beichner

Potential due to two or more charges: Superposition

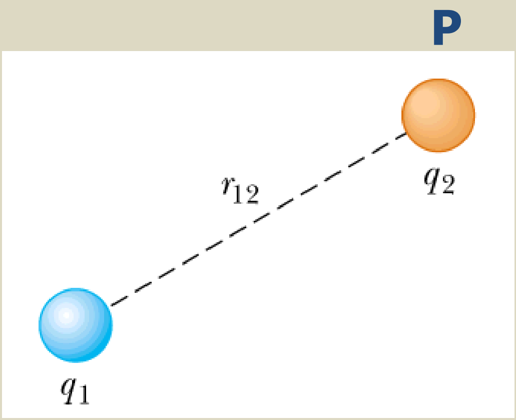
$$V = k\sum_{i}\frac{q_i}{r_i}$$



where potential is taken to be zero at infinity and r_i is the distance from the point P to the charge q_i.

Note that this is a scalar sum rather than a vector sum.

Potential Energy of a system of two charges



V_1 = potential at a point P due to q_1, external agent must do work to bring a second charge q_2 from infinity to P and this work = q_2V_1.

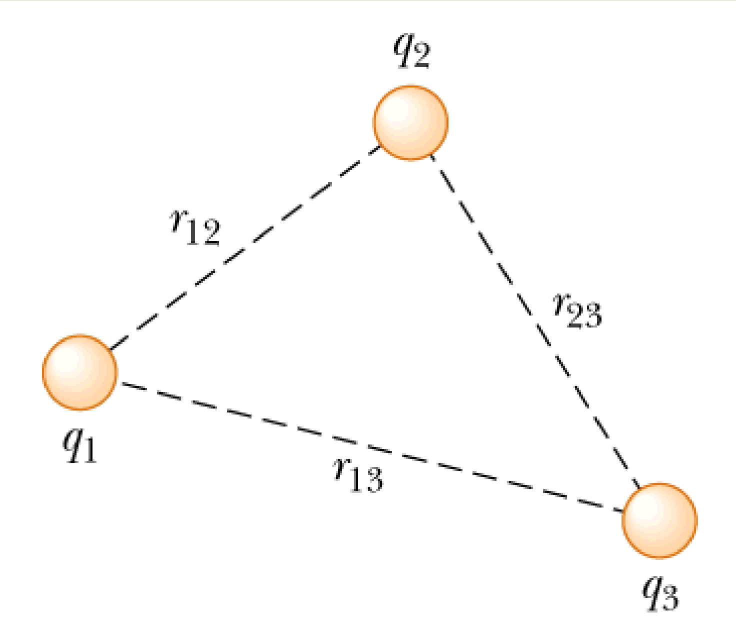
Definition: This work done is equal to the potential energy U of the two-particle system.

If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by

$$U_{12} = k\frac{q_1 \quad q_2}{r_{12}} \qquad k = \frac{1}{4\pi\varepsilon_o}$$

Pictures from Serway & Beichner

Potential Energy

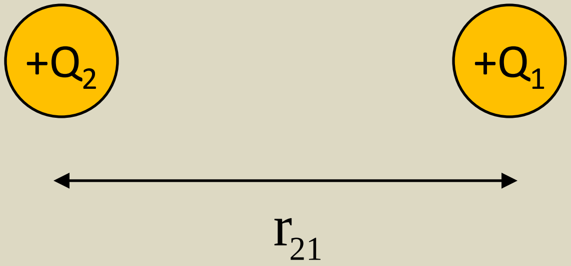


Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by

$$U = k\left(\frac{q_1 \quad q_2}{r_{12}} + \frac{q_1 \quad q_3}{r_{13}} + \frac{q_2 \quad q_3}{r_{23}}\right)$$

Pictures from Serway & Beichner

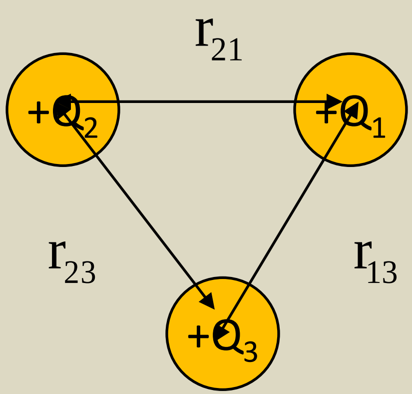
Potential energy due to multiple point charges



$$V \;\; r = \frac{kq}{r}$$

$$V = \frac{kq_1}{r_{12}}$$

$$U = q_2 V = \frac{kq_1 q_2}{r_{12}}$$



$$V = \frac{kq_1}{r_{13}} + \frac{kq_2}{r_{23}}$$

$$U = \frac{kq_1 q_2}{r_{12}} + \frac{kq_1 q_3}{r_{13}} + \frac{kq_2 q_3}{r_{23}}$$

Visualization of Electric Potentials

- The scalar electric potential can be visualized using ***equipotential surfaces***.
- An ***equipotential surface*** is a surface over which *V* is a constant.
- Because the electric field is the negative of the gradient of the electric scalar potential, the electric field lines are everywhere normal to the equipotential surfaces and point in the direction of decreasing potential.

CH-6

ELECTRIC CURRENT

Electric Charge

- Electric current = a flow of electric charges. The electrons, which orbit the nucleus at relatively large distances can sometimes become free to move → electric current.

- Electric charge is measured in coulombs (C)
- The charge on an electron = e

  $= 1.6 \times 10^{-19}$C

Current

Electric current (I)= amount of charge passing a point every second.

Measured in Amperes (A).

Measured using an ammeter, or for very small currents, a galvanometer

Current = Charge / time

$I = Q / t$

Conventional Current.

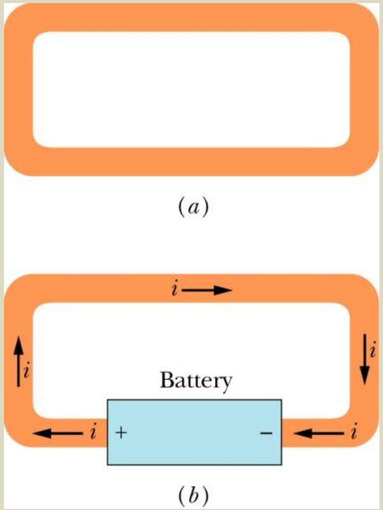
- Current = movement of electrons
- Electrons move from negative to positive
- However traditionally current was thought of as flowing from positive terminal of a battery to the negative (wrong way round)
- We still use this convention today-current is thought of as flowing from positive to negative (conventional current)

DC and AC

- Two types of current exist
- Direct current-always flows in the same direction – this is the type of current you get from a battery
- Alternating current – here the current reverses direction many times per second- this is the type of current you get from the mains

Moving charges and electric currents

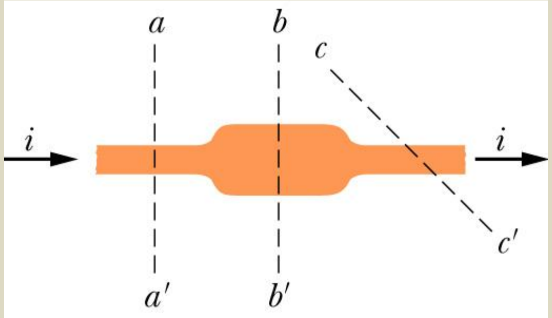
There is a current when there is a *net* flow of charges in motion.



Examples of current:

- Lightning strokes
- In neurons to regulate muscular activities.
- In conductors: in household wiring, light bulbs, and electrical appliance.
- Beam of electrons: picture tube in TV.
- Charges of both signs: ionized gases of fluorescent lamps.
- In electrolytes: car battery.
- In semiconductor chips: p-type on n-type.

ELECTRIC CURRENT



$$i = dq/dt$$

$$q = \int i \, dt$$

For steady currents:

$$q = I \, Dt$$

The SI units of current is C/s ≡ ampere: A

Electric current

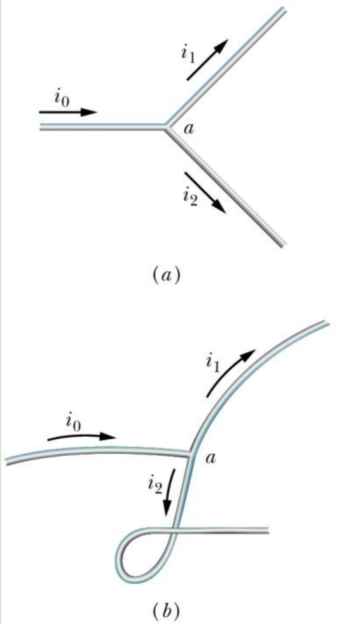
Current is a scalar quantity; the arrows in figures do not indicate vectors; they merely show direction (or sense) of flow along a conductor, not a direction in space!

Convention: (for historical reasons)

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

At a branching,

$$i_o = i_1 + i_2 + i_3 + \dots$$



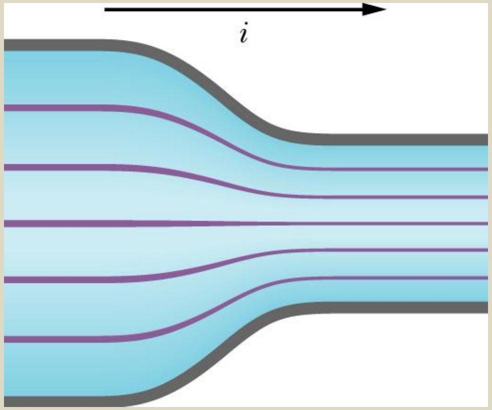
Current density (**J**)

The current density is a *vector* that describes the flow of charge through a cross section of the conductor at a particular point.

What is the direction of **J**?

J = |**J**| is the current per unit area through an element.

$$i = \int \mathbf{J} \cdot d\mathbf{A}$$



For a uniform current parallel to d**A** ,

$$J = i/A$$

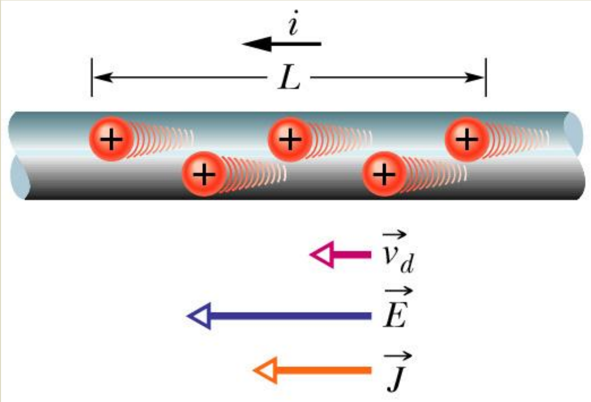
SI units for J: A/m^2

The concept of streamlines: stream lines that are closer together imply greater current density.

In figure 27-4 the current is the same for every plane that passes completely through the conductor, but the current density is not the same everywhere!

Drift speed:

When there is a current, the random speed of electrons ~ 10^6 m/s; however, the drift speed (v_d) of electrons ~ 10^{-4} m/s, in the direction opposite of the direction of the applied electric field that causes the current.



Relation between drift speed and current density

$$q = (n\,A\,L)\,e$$

$$t = L/\,v_d$$

Therefore,

$$v_d = i/(n\,A\,e)$$

or

$$\mathbf{J} = n\,e\,\mathbf{v_d}$$

Note:

|n|: is the density of charge carriers.

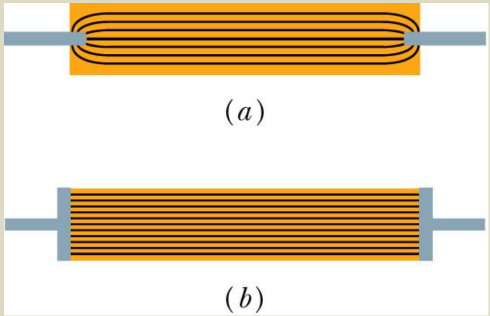
(n e): is the density of charge.

For negative charge carriers, **J** and $\mathbf{v_d}$ have opposite directions.

Resistance and Resistivity

Resistance (R) is a measure of how much an *object* resists current for a specific potential difference across its two ends.

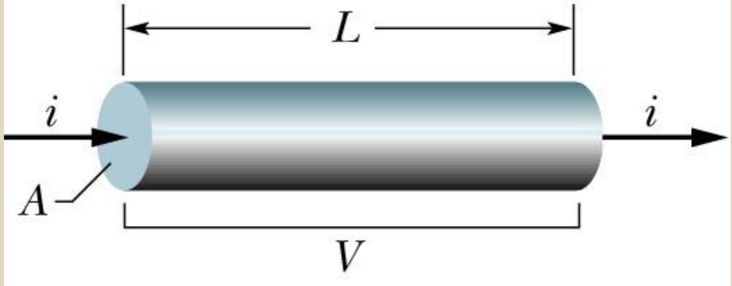
$R = V/i$



High R means little i for a specific V.

Can two objects made from the same material have different R?

Two wires of the same length, made of the same material have different cross sectional areas. Which one has a larger R?



The [R] is V/A ≡ W = ohm

A resistor is a device whose function is to provide resistance.

Resistance and Resistivity

Resistivity (r) is a measure of how much resistance a specific *material* has to current, regardless of the object's shape.

[For *isotropic* material]:

r = E/J

[r] = W m

Conductivity (s) is the inverse of r. Therefore,

J = s **E**

[s] = $(W\ m)^{-1}$

Note: $W^{-1} \equiv$ mho

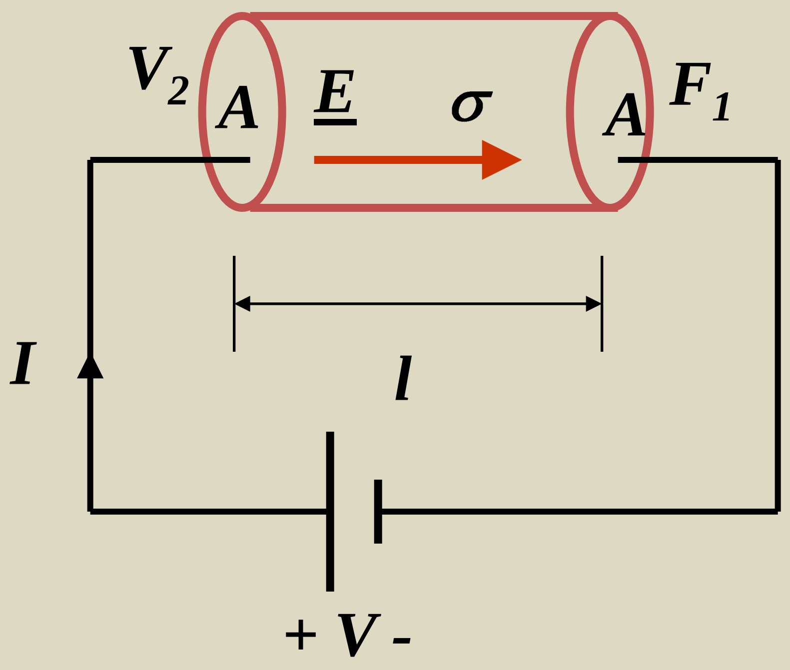
How does R relate to r for a wire of length L and cross sectional area A?

$$R = r L/A$$

Can two objects made from the same material have different r?

Ohm's Law and Resistors

- Consider a conductor of uniform cross-section:



- Let the wires and the two exposed faces of the "resistor" be perfect conductor.

- In a perfect conductor:
  - ***J*** is finite
  - σ is infinite
  - ***E*** must be zero.

Ohm's Law and Resistors (Cont'd)

- To derive Ohm's law for resistors from Ohm's law at a point, we need to relate the circuit quantities (V and I) to the field quantities ($\mathbf{E}$ and $\mathbf{J}$)
- The electric field within the material is given by

$$E = \frac{V_{12}}{l} = \frac{V_2 - V_1}{l} = \frac{V}{l}$$

- The current density in the wire is

$$J = \frac{I}{A}$$

Ohm's Law and Resistors (Cont'd)

- Plugging into ***J* = *sE***, we have
- Define the resistance of the device as

$$V = \frac{l}{\sigma A} I$$

$$R = \frac{l}{\sigma A}$$

$$V = RI$$

Ohm's law for resistors

Ohm's law

Ohm's law is not *really* a "law".

It is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

So, devices can be categorized into Ohmic or Non-Ohmic.

If, for V = i R, the resistance (R) is constant independent of V then we say the device/ material is Ohmic.

Similarly, if r is constant [for E = r J] the material is Ohmic.

In general, conductors are Ohmic as long as the electric field is not too high.

Electromotive force (emf)

- EMF = the voltage between two ends of a circuit when no current is flowing in the circuit

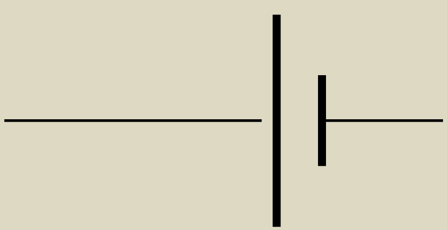
Sources of EMF

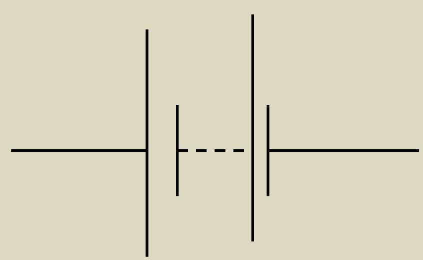
1. Electric cells-convert chemical energy to electrical energy

Consists of 2 different metals (the electrodes) immersed in a substance called an electrolyte.

A battery consists of a no. of cells connected together (a car battery = 6 2V cells in series)

Primary cell



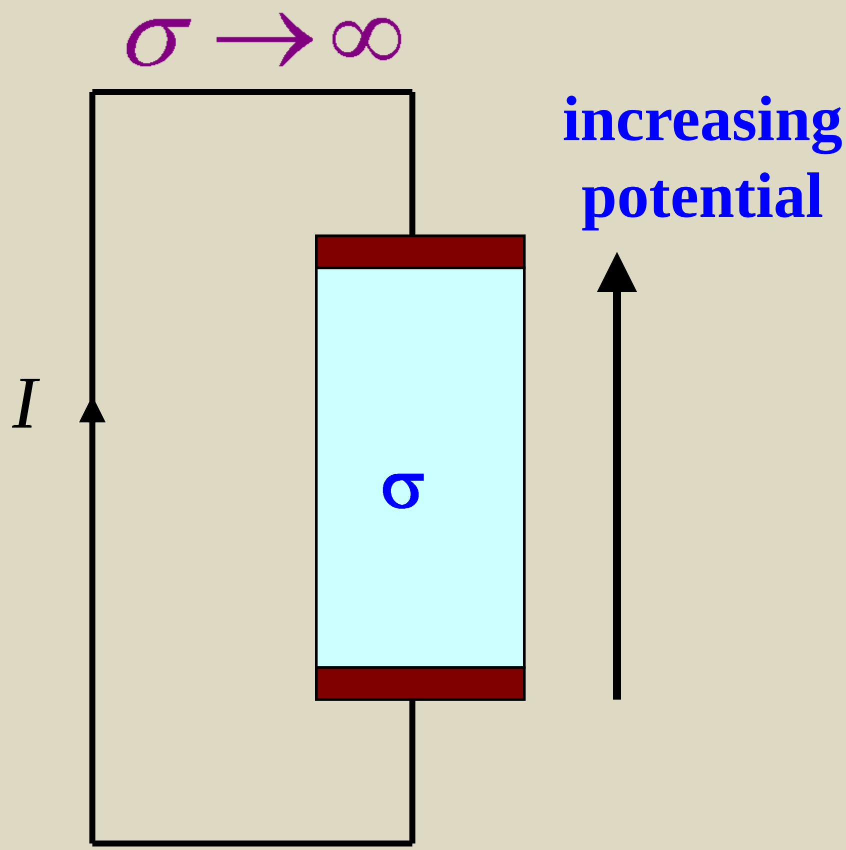


Battery of cells

Electromotive Force

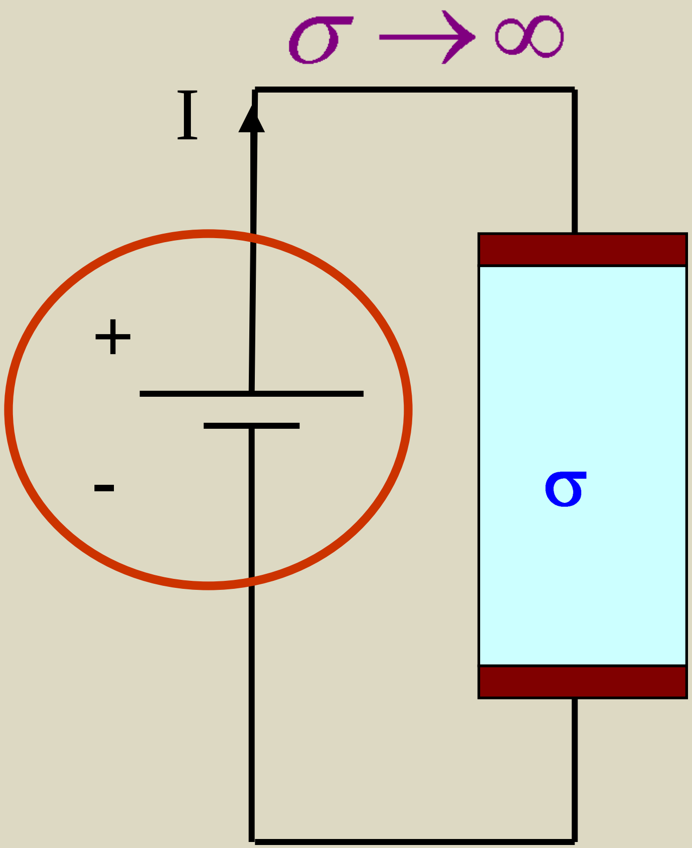
- Steady current flow requires a closed circuit.
- Electrostatic fields produced by stationary charges are conservative. Thus, they cannot by themselves maintain a steady current flow.

Electromotive Force (Cont'd)



- The current *I* must be zero since the electrons cannot gain back the energy they lose in traveling through the resistor.

Electromotive Force (Cont'd)

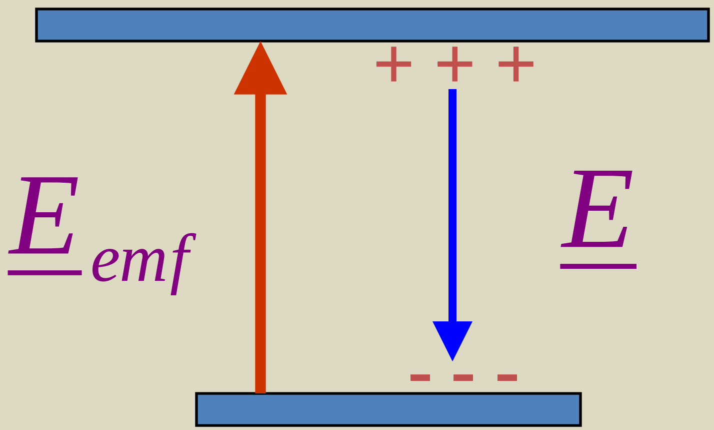


- To maintain a steady current, there must be an element in the circuit wherein the potential ***rises*** along the direction of the current.

Electromotive Force (Cont'd)

- For the potential to rise along the direction of the current, there must be a ***source*** which converts some form of energy to electrical energy.
- Examples of such sources are:
  - **batteries**
  - **generators**
  - **thermocouples**
  - **photo-voltaic cells**

Inside the Voltage Source



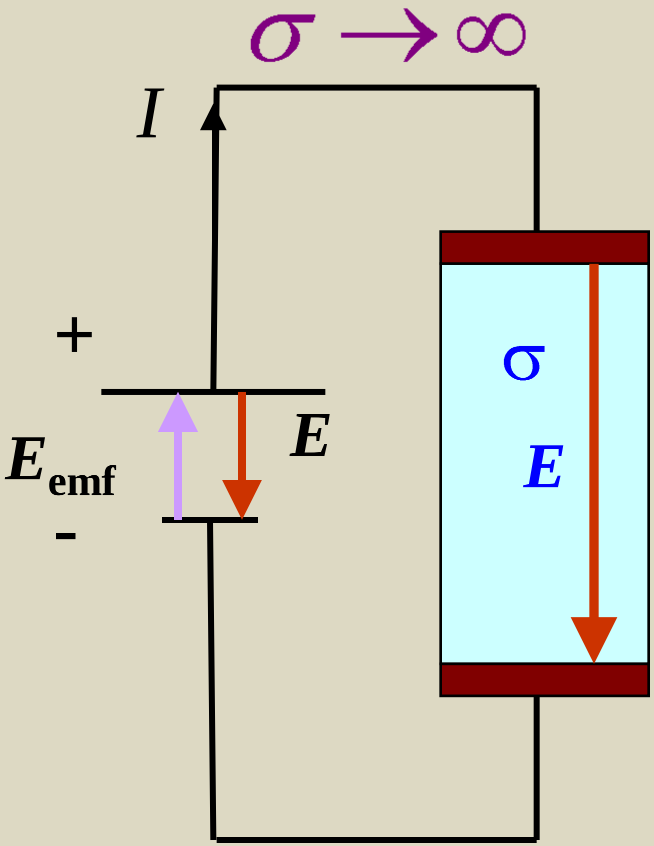
- $\boldsymbol{E}_{emf}$ is the electric field established by the energy conversion.
- This field moves positive charge to the upper plate, and negative charge to the lower plate.
- These charges establish an electrostatic field $\boldsymbol{E}$.

In equilibrium:

$$\underline{E}_{emf} + \underline{E} = 0$$

Source is not connected to external world.

Electromotive Force (Cont'd)



At all points in the circuit, we must have

$$\frac{\underline{J}}{\sigma} = \underline{E}_{total} = \underline{E}_{emf} + \underline{E}$$

exists only in battery

Electromotive Force (Cont'd)

- Integrate around the circuit in the direction of current flow

$$\oint_C \underline{E}_{total} \cdot d\underline{l} = \oint_C \frac{1}{\sigma} \underline{J} \cdot d\underline{l}$$

$$\oint_C \underline{E} \cdot d\underline{l} + \int_{-}^{+} \underline{E}_{emf} \cdot d\underline{l} = \oint_C \frac{1}{\sigma} \underline{J} \cdot d\underline{l}$$

0

Electromotive Force (Cont'd)

- Define the ***electromotive force*** (*emf*) or "voltage" of the battery as

$$V_{emf} = \int_{-}^{+} \underline{E}_{emf} \cdot d\underline{l}$$

Electromotive Force (Cont'd)

- We also note that

- Thus, we have the circuit relation

$$\oint_C \frac{1}{\sigma} \underline{J} \cdot d\underline{l} = \frac{l}{\sigma A} I = RI$$

$$V_{emf} = RI$$

Continuity Equation

- Using the *divergence theorem*, we have

$$\oint_S \underline{J} \cdot d\,\underline{s} = \int_V \nabla \cdot \underline{J}\, dv$$

- We also have

$$\frac{d}{dt}\int_V q_{ev}\, dv = \int_V \frac{\partial q_{ev}}{\partial t}\, dv$$

Becomes a partial derivative when moved inside of the integral because q_{ev} is a function of position as well as time.

Continuity Equation (Cont'd)

- Thus,

$$\int_V \nabla \cdot \underline{J}\, dv + \int_V \frac{\partial \rho}{\partial t}\, dv = 0$$

- Since the above relation must be true for *any and all regions*, we have

$$\nabla \cdot \underline{J} + \frac{\partial \rho}{\partial t} = 0$$

← Continuity Equation

Continuity Equation (Cont'd)

- For steady currents,

$$\frac{\partial \rho}{\partial t} = 0$$

- Thus,

$$\nabla \cdot \underline{J} = 0$$

J is a *solenoidal* vector field.

Continuity Equation in Terms of Electric Field

- Ohm's law in a conducting medium states

$$\underline{J} = \sigma \underline{E}$$

- For a homogeneous medium

$$\nabla \cdot \underline{J} = \sigma \nabla \cdot \underline{E} = 0 \quad \Rightarrow \quad \nabla \cdot \underline{E} = 0$$

- But from Gauss's law,

$$\nabla \cdot \underline{E} = \frac{q_{ev}}{\varepsilon}$$

- Therefore, the volume charge density, ρ, must be zero in a homogeneous conducting medium

Power in electric circuits

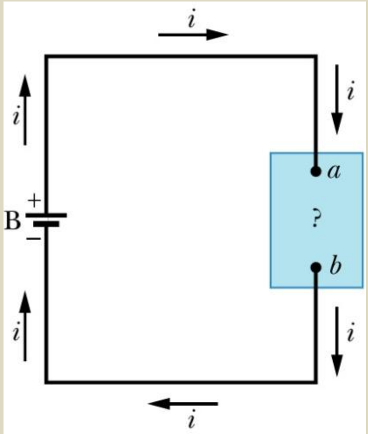
From the principle of conservation of energy, the transfer of energy is equal to the decrease in electric potential energy in going from [a] to [b].

The rate of such transfer is:

$$P = i V$$

The energy may be transferred into a resistor, motor, rechargeable battery, …etc.

What is the rate of transfer of energy from the battery to the device? (the same P!)



Power in electric circuits

If the ‘device’ is a resistor, energy will be “dissipated”. Why?

For Ohmic devices, the thermal energy produced (i.e. dissipated) is:

$$P = i^2 R = V^2/R$$