Special Theory of Relativity

A Brief introduction

Classical Physics

- At the end of the 19th century it looked as if Physics was pretty well “wrapped up”.

- Newtonian mechanics and the law of Gravitation had explained how the planets moved and related that to how ordinary objects here on earth responded to forces.

Classical Physics (Cont)

- Kinetic theory explained the behavior of gases.

- Maxwell's Theory of Electromagnetism explained the phenomena of electricity and magnetism, predicted electromagnetic waves and identified light as an example of same.

Classical Physics (Cont)

- All this came to be known as **classical physics**.

- Little did the physicist of 1900 realize what was in store during the next 100 years, when the ideas, theories, and results of **modern physics** were developed.

Twentieth Century Physics

- Special Theory of Relativity
- General Theory of Relativity
- Quantum Theory

Special Theory of Relativity

Introduced a new way to view

- Space
- Time
- Simultaneity

General Theory of Relativity

- Re-interpreted gravitational theory in terms of space-time.

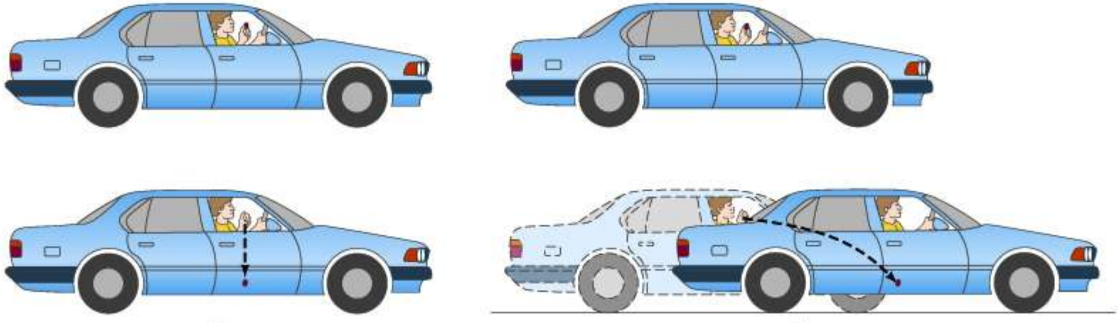
Quantum Theory

- Introduced a new way to think about atomic processes
  - Replaced “absolute knowledge” with probabilities
- Helped “clear up” some problems that classical theories could not explain.

Galilean-Newtonian Relativity

- Inertial Reference Frame
  - Inertial reference frames are those in which Newton’s laws of motion are valid.
- Relativity Principle
  - The basic laws of physics are the same in all inertial reference frames.
- Both understood by Galileo and Newton

Galilean-Newtonian Relativity



(a)
Reference frame = car

(b)
Reference frame = Earth

Galilean-Newtonian Relativity

- Straight vertical path in the car.
- Parabolic path when reference frame is the earth.
- The laws are the same, but the paths are different because of different initial conditions.
- But both observers would agree and understand the situation.

Galilean-Newtonian Relativity

- All inertial reference frames are equivalent for the description of mechanical motion.
    - There is no test or experiment you can do to “prove” which frame is really at rest or moving with constant velocity.

Enter Maxwell and His Equations

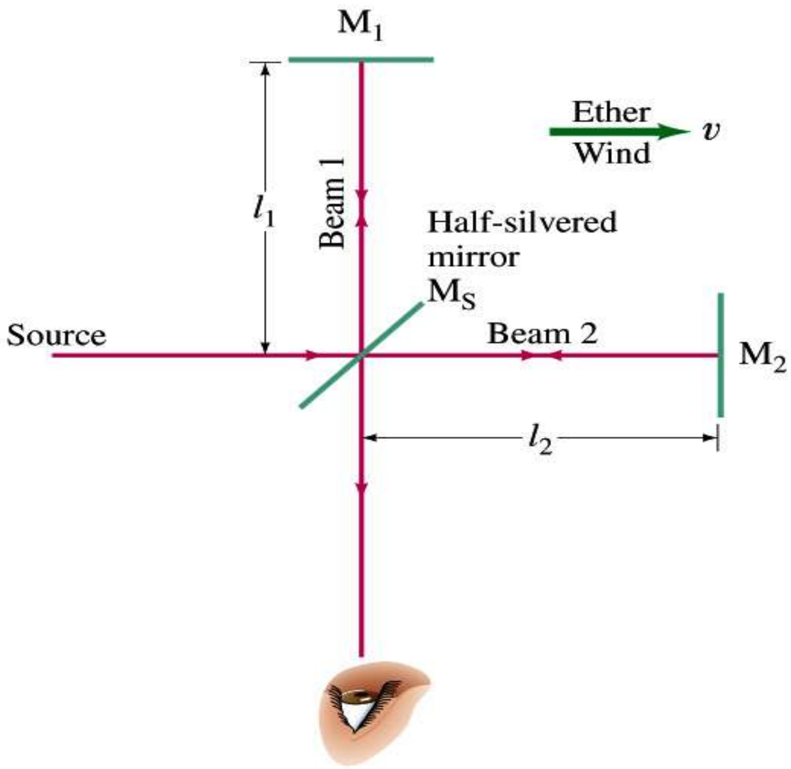
- **PROBLEM!** Maxwell's theory predicts that the velocity of light is 3×10^8 m/s and this is what is measured.

- **QUESTION!!** In what reference frame does light have this velocity?

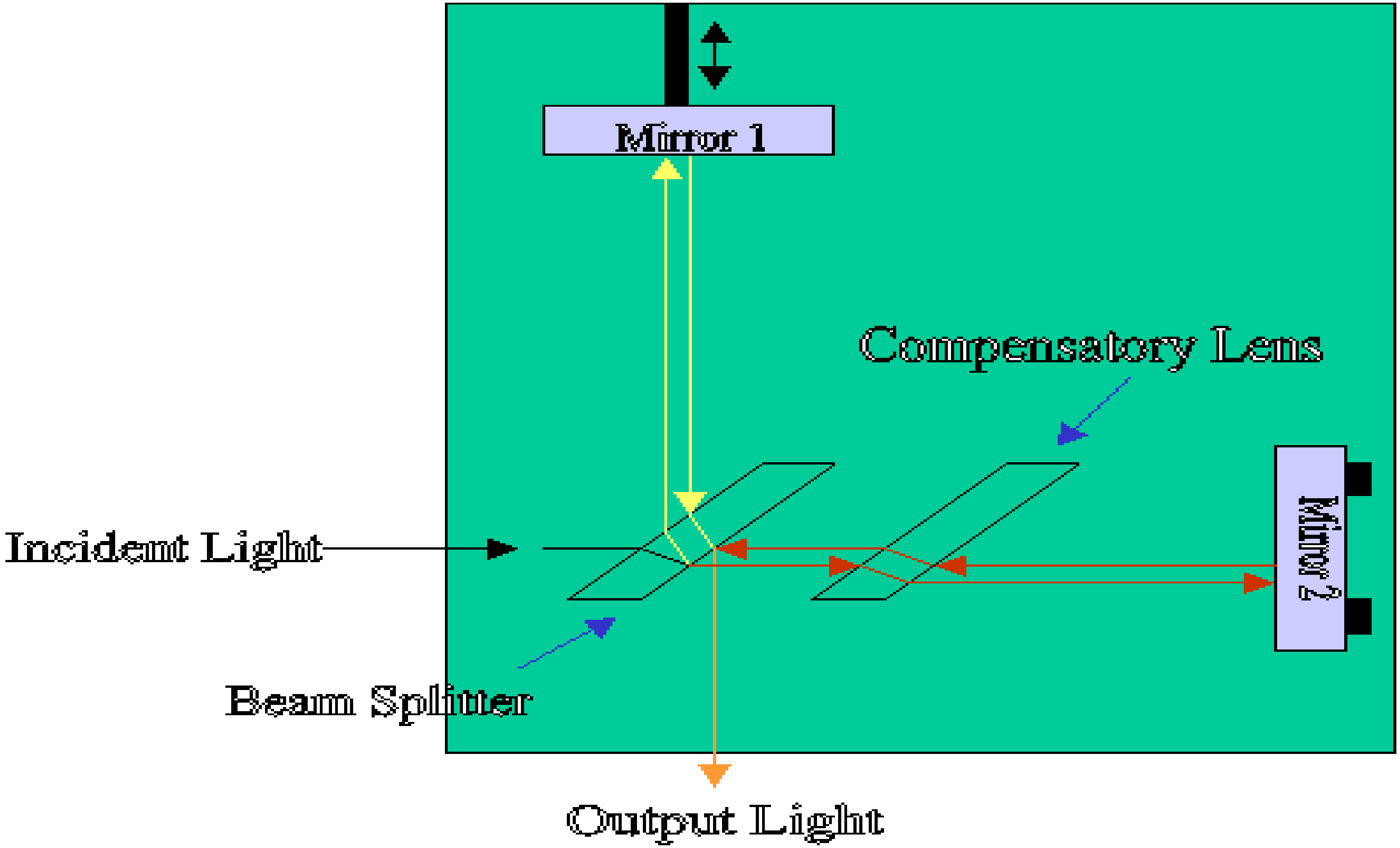
Enter Maxwell and His Equations

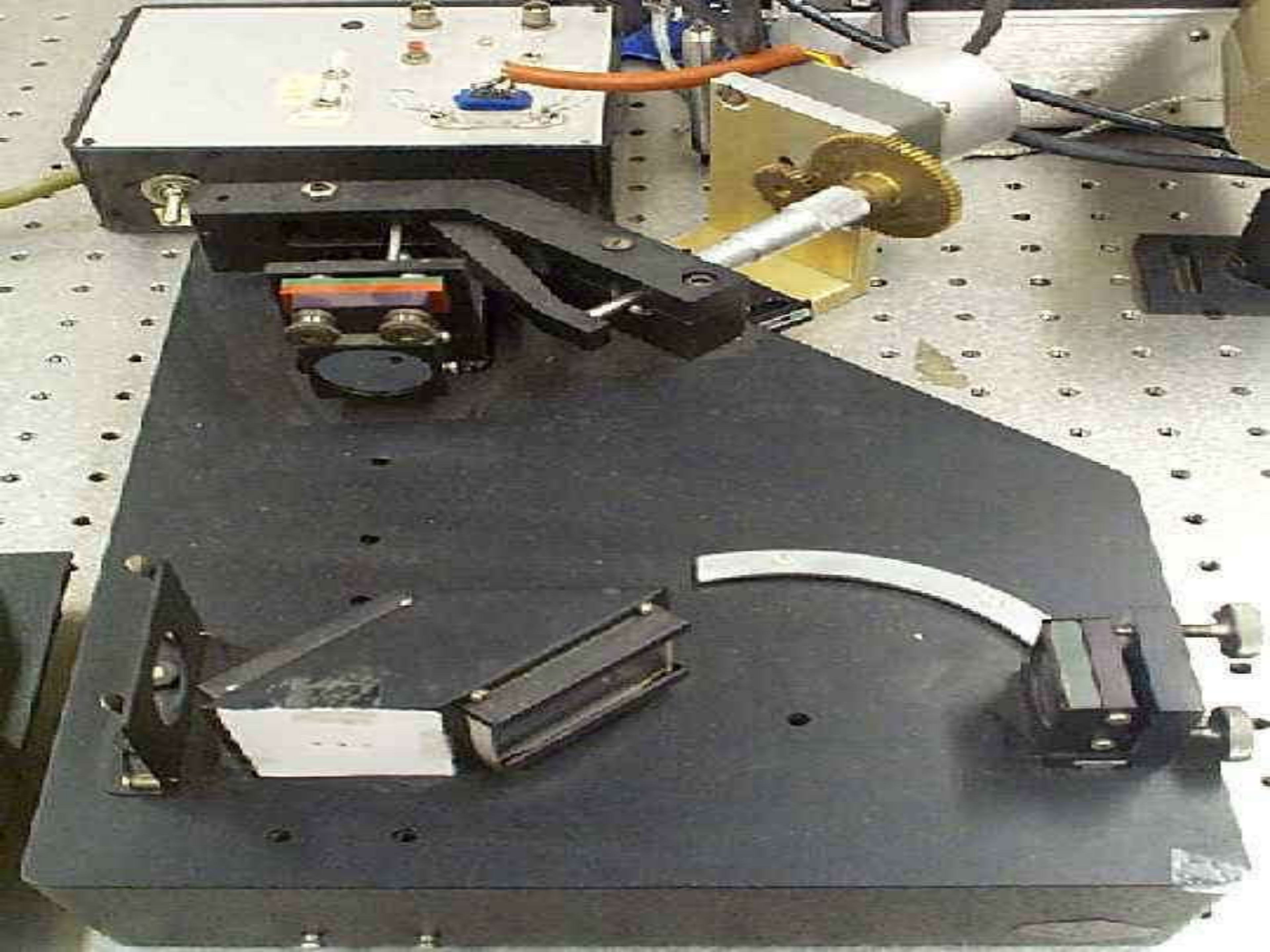
- Maxwell's Equations **did not obey** the Relativity Principle.
- They were not the same in all inertial reference systems
    - The form of the equations changes in a moving system.
    - They were the simplest in a reference frame at rest wrt the ether.
    - This implied that perhaps there was a reference frame which was at absolute rest and hence the preferred reference system.

The Michelson-Morley Experiment



The Interferometer





The Michelson-Morley Experiment

- This experiment was designed to detect the speed of the earth through the ether.
- The earth's speed around the sun is $3x10^4$m/s.
- Predicted 0.4 fringe shift

The Michelson-Morley Experiment

- Their apparatus was capable of measuring a shift of 0.01 fringe
- **NO FRINGE SHIFT WHATSOEVER** was ever detected**!!**

Explanation of the negative result:

- The result showed that speed of light is same for all observers which is not true for waves that need a material medium to occur , this is principle of constancy of speed of light.
- Lorentz gave an explanation : acc to him the size of a moving body is altered due to its motion through stationary ether.
- The moving earth drags the ether with it. Hence there is no relative motion between the two so that no shift is observed.

What to do?

- Are Maxwell's equations wrong?
  - They correctly predicted so many observations that physicists were reluctant to give them up.
- Ether is "dragged" along by the earth?
  - Got the same results when the M&M experiment was carried out in balloons and on mountaintops.
    - Each attempt to determine a way to find a preferred reference system seemed to be doomed to failure

There is a Way Out of This Mess

- **Henri Poincare** finally concluded that such a complete "conspiracy of nature" must itself be regarded as a law of nature. i.e., the Principle of Relativity must be valid!!

- This was the state of affairs in 1905 when Einstein presented his Theory of Relativity.

Frames of Reference.

Inertial frame of reference

- In which all law of motion are valid

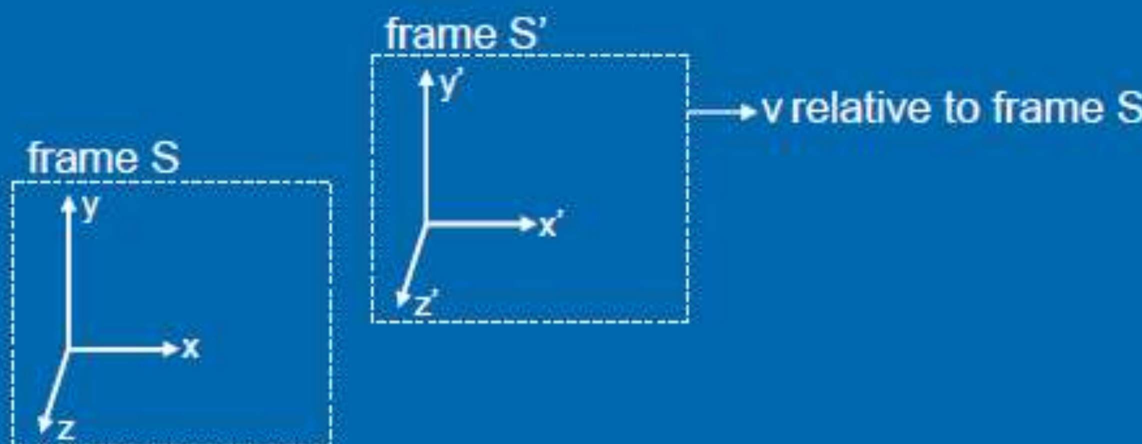
Non inertial frame of reference

- Newton law are not valid.

Galilean transformation:

- The equation relating the coordinates of a particle in two inertial frames(whose relative velocity is negligible in comparison to the speed of light) are called ***GALILEAN TRANSFORMATION.***

Transformation between frames



<u>Galilean transformations</u>

$$
\begin{aligned}
x' &= x - vt & & & v'_x &= v_x - v \\
y' &= y & &\Rightarrow & v'_y &= v_y \\
z' &= z & & & v'_z &= v_z
\end{aligned}
$$

Does not agree with the postulates of special relativity, since it implies:

$$c' = c - v$$

Need new set of transformation rules!

Enter Einstein - 1905

- In 1905 Albert Einstein proposed that we accept the fact that the speed of light was the same in all reference systems
  - (this was consistent with the M&M result) and was tantamount to doing away with the concept of the ether.

Postulates of the Special Theory of Relativity

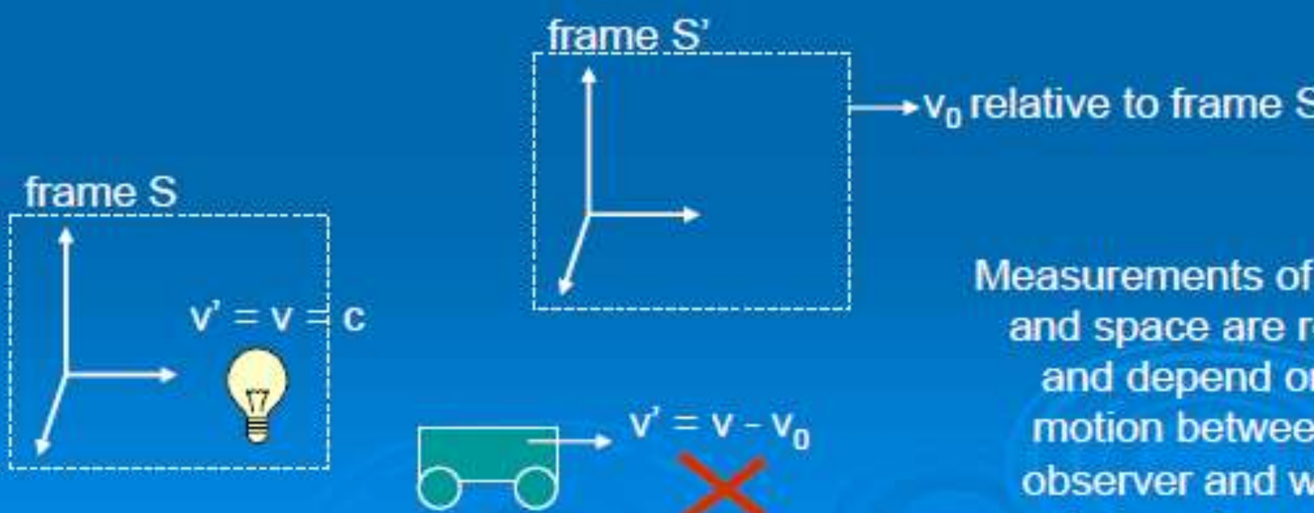
- **First Postulate**: *The laws of physics have the same form in all inertial reference systems*. (This is the Relativity Principle)

- **Second Postulate**: *Light propagates through empty space with a definite speed "c" independent of the speed of the source or of the observer.* (Agrees with experiment)

Principles of Special Relativity

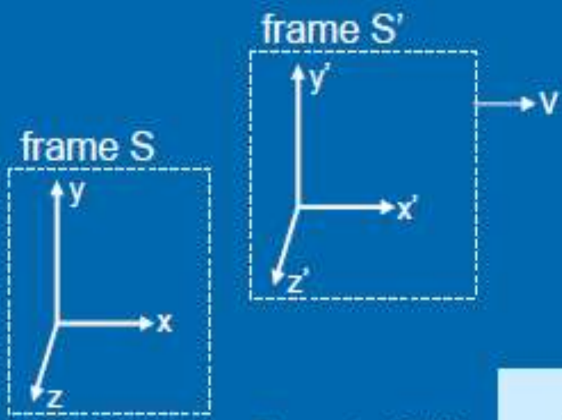
1. The laws of physics are the same in all inertial frames of reference, i.e. there is no universal frame of reference

2. The speed of light in free space has the same value in all inertial frames of reference: $2.998 * 10^8$ m/s



Measurements of time and space are relative and depend on the motion between the observer and what is being observed.

Lorentz Transformation



Assume a form: $\boxed{\begin{aligned} x' &= k(x - vt) \\ y' &= y \\ z' &= z \end{aligned}}$ (a)

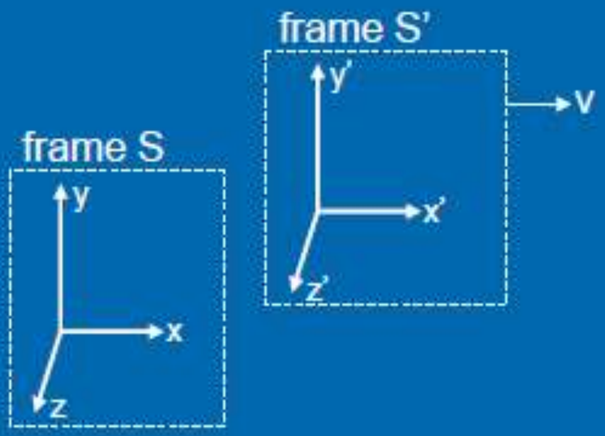
We must also have: $\boxed{x = k(x' + vt')}$ (b)

From (a)&(b): $\boxed{x = k^2(x - vt) + kvt'}$ or $\boxed{t' = kt + \left(\frac{1-k^2}{kv}\right)x}$

Can evaluate k using 2nd postulate of Special Relativity: for a light beam we have $\boxed{x = ct}$ and $\boxed{x' = ct'}$.

Using these relations we find: $\boxed{k = \frac{1}{\sqrt{1 - v^2/c^2}}}$

Lorentz Transformation



Lorentz transformation:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$
$$y' = y$$
$$z' = z$$
$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

Inverse Lorentz transformation:

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$
$$y = y'$$
$$z = z'$$
$$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

Velocity addition?

$$V_x = \frac{dx}{dt} = \frac{dx' + vdt'}{\sqrt{1 - v^2/c^2}} \frac{\sqrt{1 - v^2/c^2}}{dt' + vdx'/c^2}$$
$$= \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}} = \frac{V_x' + v}{1 + \frac{vV_x'}{c^2}}$$

Reference Frames

The frame of reference for a person or object is the coordinate system which moves with the person or with the object.

Think of the frame of reference as a set of *xyz* axes which are attached to the object.

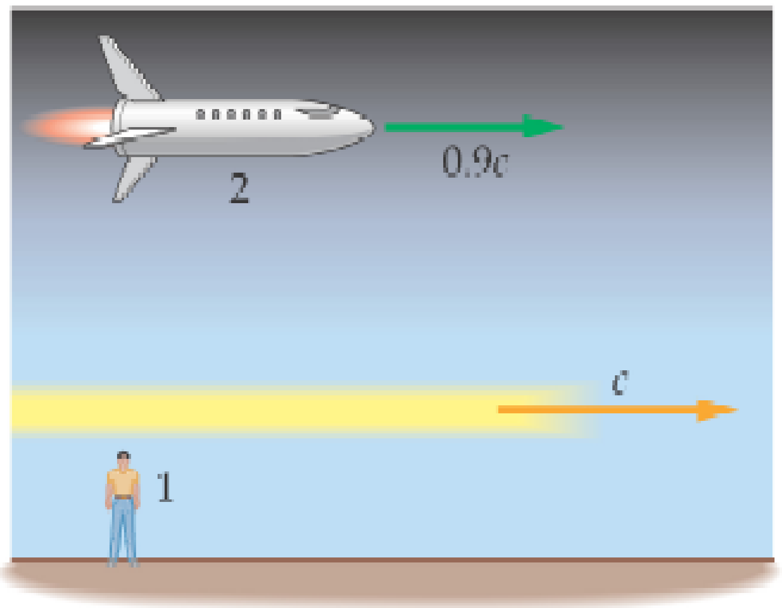
An inertial reference frame is one in which Newton's Laws are valid. All inertial reference frames move with constant velocity relative to one another.

The Speed of Light

If we apply Newtonian Mechanics to describe the propagation of light, then two observers who are in two different reference frames will measure a different value for the speed of light if there is relative motion between the reference frames. (Recall $v_{13} = v_{12} + v_{23}$)

However, that does not happen – all observers measure the same speed of light.

We need Relativity Theory to explain this!



In Relativity Theory, the distance between two points and the time interval between two events depend on the frame of reference in which they are measured!

There is no absolute measure of time.

There is no absolute measure of length.

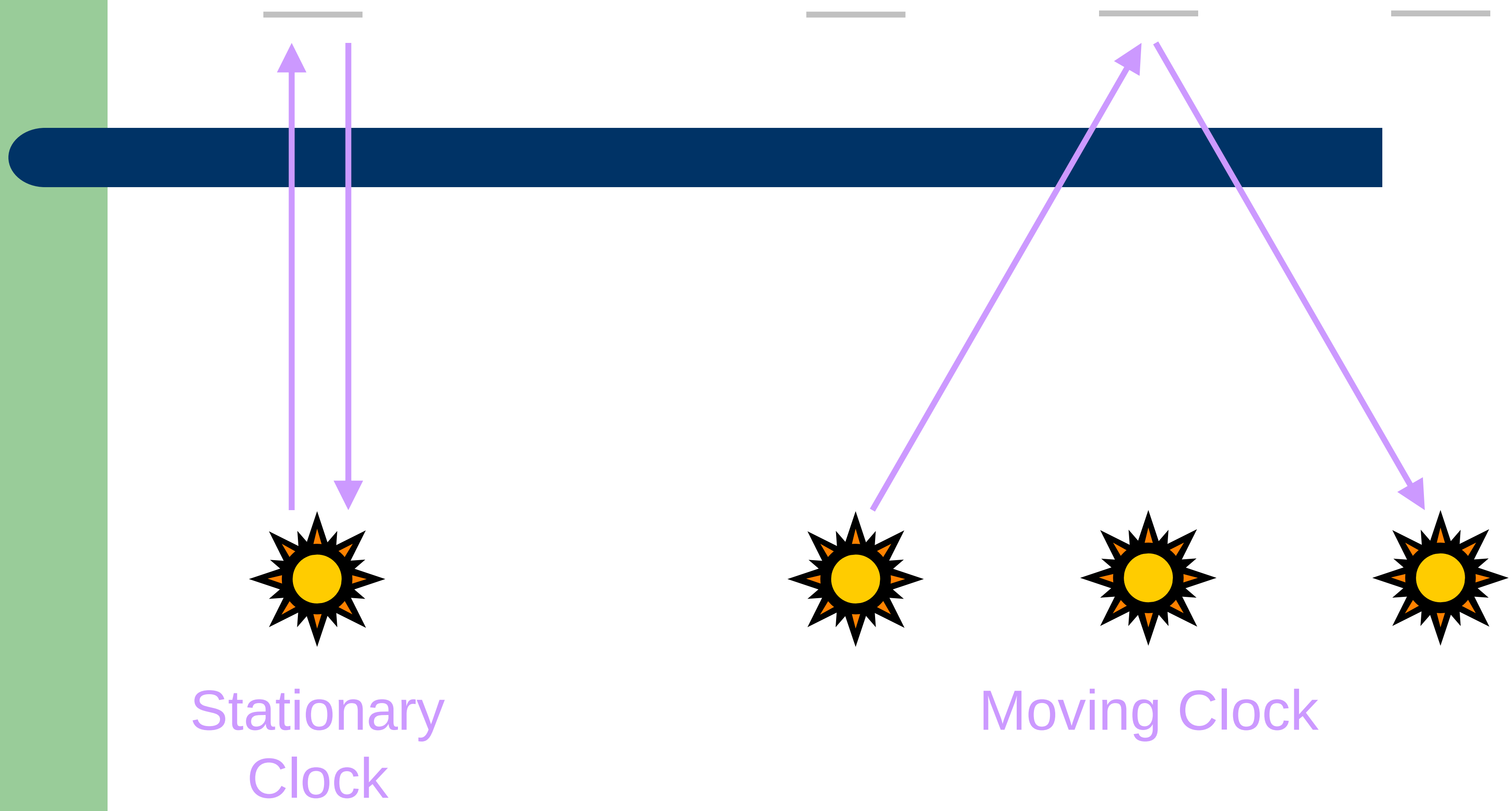
Time is Relative

Time interval measurements depend on the frame in which they are measured.

Two events which appear to be simultaneous in one reference frame are in general not simultaneous in a second frame moving with respect to the first.

Measuring time on a moving clock:

Time Dilation



A moving clock runs <u>slower</u> than its counterpart at rest

Time Dilation

Two observers, each in their own reference frame moving with a relative velocity will not agree on how fast time passes. Each will think the other's clock is wrong.

Moving clocks run slow

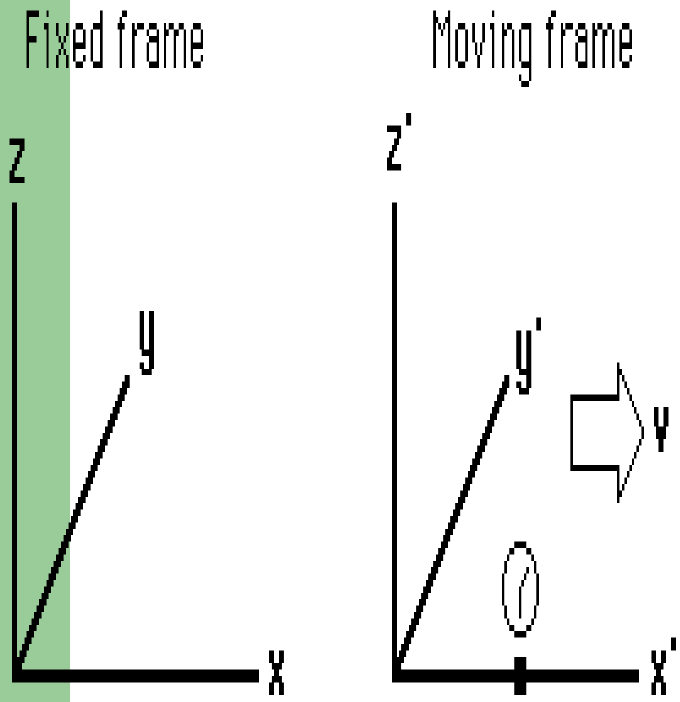
This effect is known as time dilation.

The proper time Δt_0 is the time interval between two events as measured by an observer who sees the events occur at the same place. Let the "proper" frame move with velocity *v* w.r.t. another frame.

Time interval in moving frame: $\Delta t = \gamma \Delta t_0$

where $$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

Time Dilation



If the time interval $T_0 = t_2' - t_1'$ is measured in the moving reference frame, then $T = t_2 - t_1$ can be calculated using the Lorentz transformation.

$$T = t_2 - t_1 = \frac{t_2' + \frac{vx_2'}{c^2} - t_1' + \frac{vx_1'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

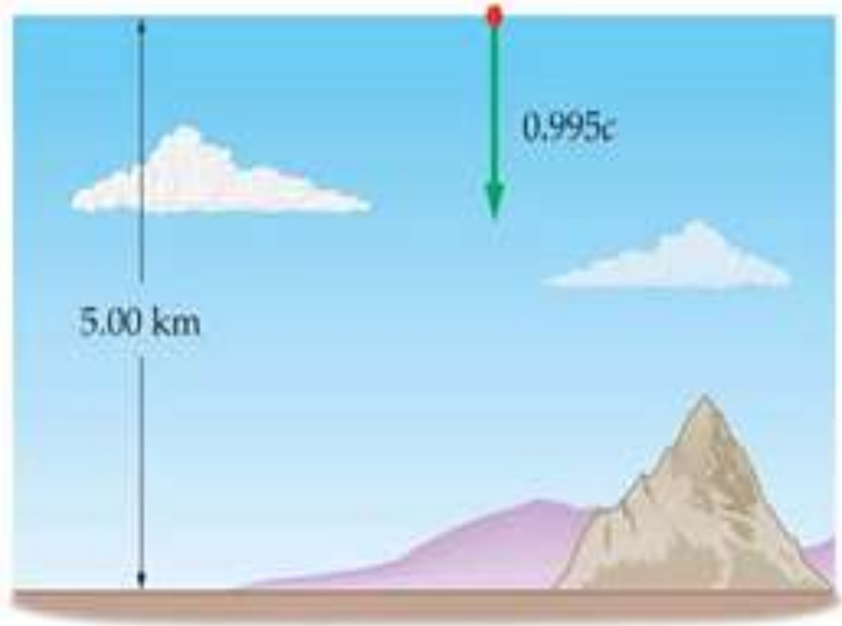
The time measurements made in the moving frame are made at the same location, so the expression reduces to:

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} = T_0 \gamma$$

Time dilation has been measured with extremely accurate atomic clocks in airplanes, and also is frequently observed in subatomic particles.

Another consequence of time dilation is that different observers will disagree about the simultaneity of events occurring at different places.

The Life and Times of a Muon



Definitions:

Event: a physical occurrence that happens at a specified location at a specified time.

Proper time: the amount of time separating two events that occur at the same location.

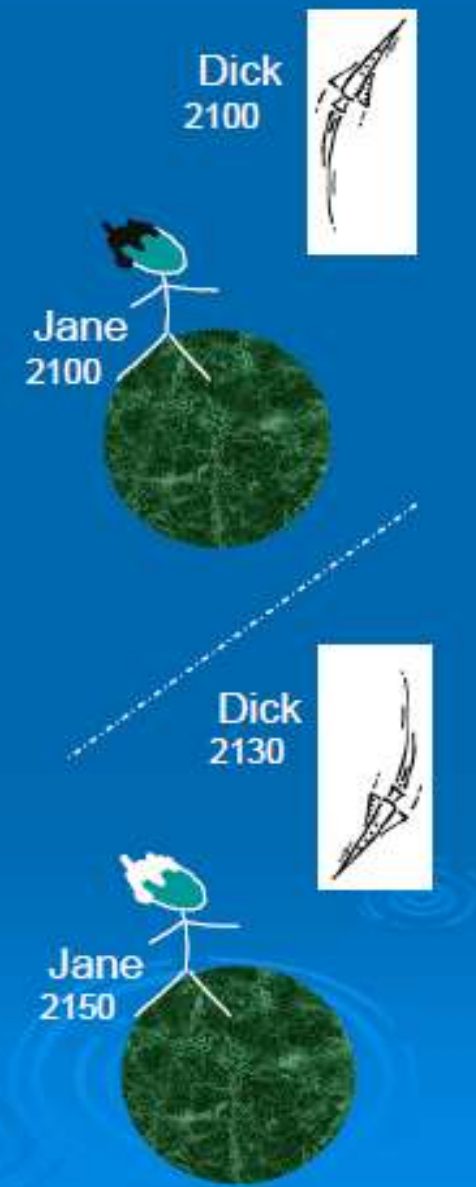
Twin Paradox

In Dick's frame of reference the distance he has traveled is less than what Jane measures:

$$L = L_0\sqrt{1 - v^2 / c^2} = 0.6L_0$$

Therefore the time it takes Dick to travel this distance is shorter than what Jane measures. So despite Jane's 50 year wait Dick only spends 30 years traveling and when he returns he is 20 years younger than Jane!

* A clock that leaves an inertial system and returns to it after moving relative to that system, will always find itself slow compared to the clocks that stayed in the system.



Length Contraction

The distance between two points depends on the frame of reference in which it is measured.

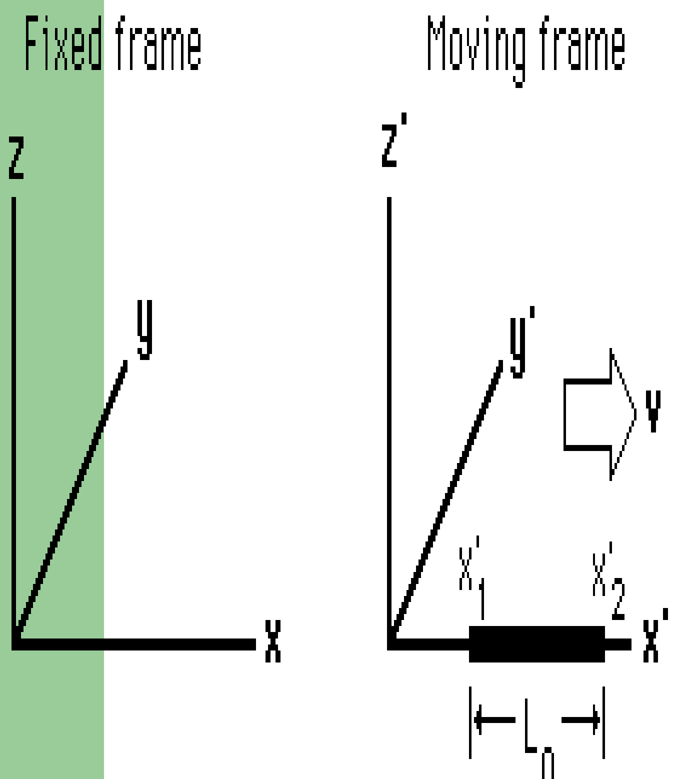
The proper length L_0 is the length of the object measured in the frame of reference in which the object is at rest.

If the object is moving in a reference frame, its length L will be measured to be *less* than its proper length:

$$L = L_0 / \gamma = L_0 \sqrt{1 - v^2/c^2}$$

This is known as relativistic length contraction.

Length Contraction



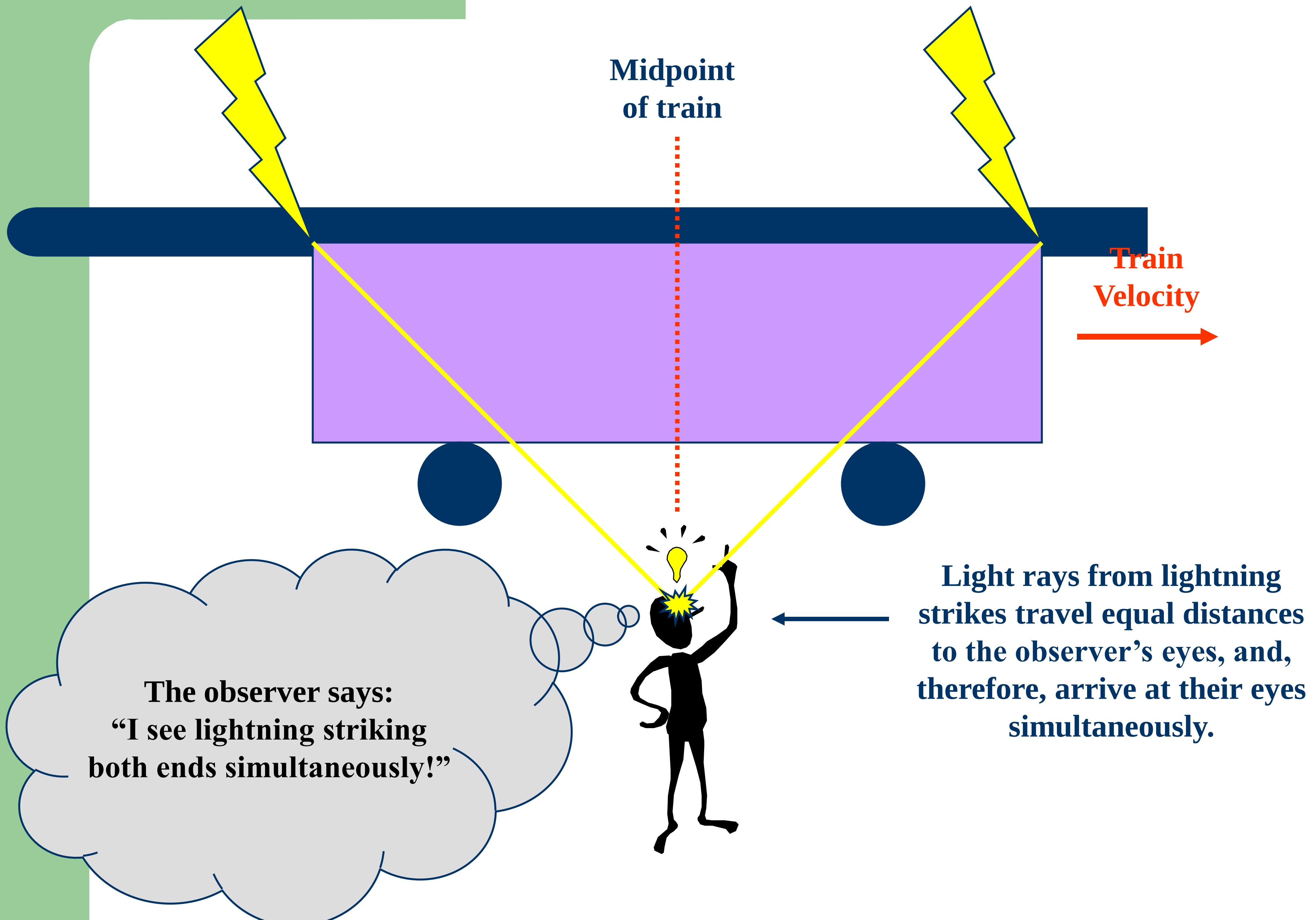
If the length $L_0 = x'_2 - x'_1$ is measured in the moving reference frame, then $L = x_2 - x_1$ can be calculated using the Lorentz transformation.

$$L_0 = x'_2 - x'_1 = \frac{x_2 - vt_2 - x_1 + vt_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

But since the two measurements made in the fixed frame are made simultaneously in that frame, $t_2 = t_1$, and

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$

Length contraction


Midpoint
of train
Train
Velocity
The observer says:
"I see lightning striking
both ends simultaneously!"
Light rays from lightning
strikes travel equal distances
to the observer's eyes, and,
therefore, arrive at their eyes
simultaneously.

The observer on the train is carried away from the spot where lightning struck the rear of the train. Therefore, the light ray from the rear has more distance to travel and arrives at the observer's eye later than the light ray from the front.

During the train's forward movement the observer on the train is carried towards the spot where the lightning struck the front of the train. Therefore, the light ray from the front has less distance to travel to the observer's eye and arrives sooner than the light ray from the rear.

So what?

Pre-Einstein: the person on the train would say that the lightning strikes still occurred simultateously.

[redacted] that I rush towards the light coming from the front and away from the light at the back….

But…and it is a massive but…..

They are NOT allowed to say “I rush away from light” or “I rush towards light”.

For <u>both</u> observers the speed of light travels at the <u>same</u> speed (300 million metres per second).

The second observer is therefore <u>not</u> allowed to say: “the lightning really strikes both ends of the train at the same time.” They <u>must</u> conclude that the lightning hit the front before it hit the back!

The implication is that what is simultaneous to one person need not be simultaneous to someone else

This is very weird…it means that the notion of "at the same time" is subjective.

And it is NOT a trick of light…you can imagine that the lightning strikes hit clocks located at the site of the strikes, causing them to stop.

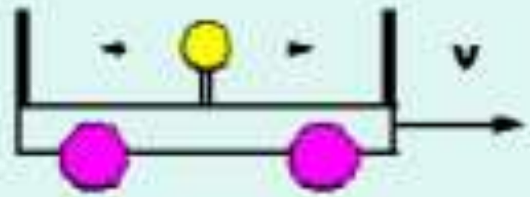
Clocks on the train at the front and back are stopped at different times.

Whilst two clocks at rest on the platform but still adjacent to the lightning strikes record the same time.

his immediately implies that time is <u>NOT</u> universal.

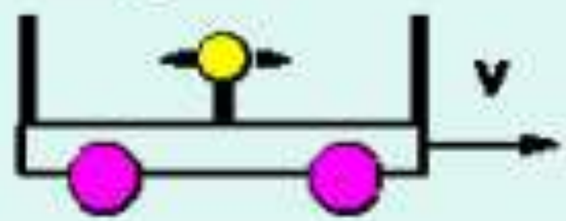
Simultaneity

Light in center of car flashes, hits front and back simultaneously.

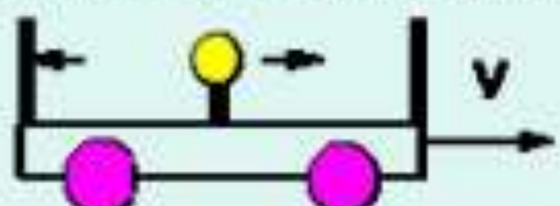


To an observer watching the car move, the car travels while the light is in motion. So the light hits the back of the car before the front.

Flash goes off



Flash reaches back wall first

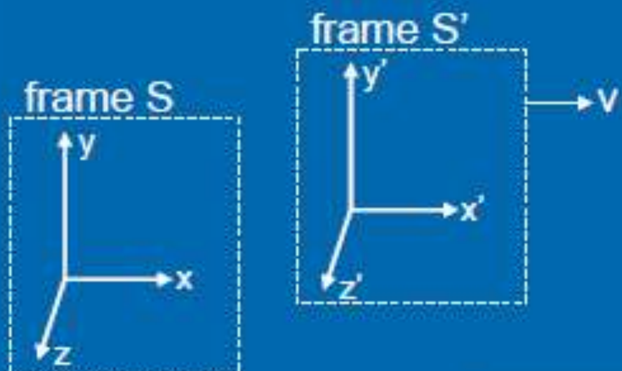


Simultaneity?

- Which conclusion is correct?
- Einstein said that we must regard both answers as being correct.

Relativistic Momentum

Is p = mv a valid definition for momentum in inertial frames in relative motion?



Particle A is at rest in frame S And B is at rest in frame S'.

At the same instant A and B are thrown in opposite directions in y with speeds V_A and V'_B.

In their rest frames: $$m_A = m_B \qquad V_A = V'_B$$

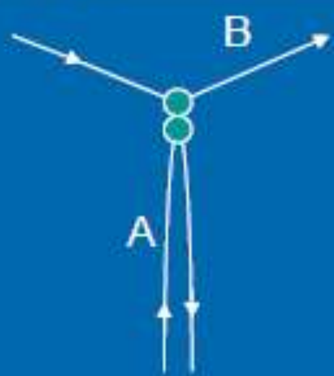
And they each travel a distance Y/2 before they collide.

The round trip time in S and S' is $T_0 = Y / V_A = Y / V'_B$

In S the speed of B is measured as $$V_B = \frac{Y}{T} = \frac{Y}{T_0}\sqrt{1 - v^2 / c^2} = V_A\sqrt{1 - v^2 / c^2}$$

Relativistic momentum

Collision as seen from frame S



$$V_B = V_A \sqrt{1 - v^2 / c^2}$$

$$p_A = m_A V_A$$

$$p_B = m_B V_B = m_B \sqrt{1 - v^2 / c^2}\, V_A$$

Momentum is conserved only if $$m_B = \frac{m_A}{\sqrt{1 - v^2 / c^2}}$$

We can define mass of a moving particle in terms of its rest mass as $$m(v) = \frac{m}{\sqrt{1 - v^2 / c^2}}$$ rest mass or proper mass

So the relativistic momentum is $$p = \frac{mv}{\sqrt{1 - v^2 / c^2}} = \gamma m v$$ with $$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

Mass and Energy

$$\boxed{E_0 = mc^2}$$

Lets calculate the work done on a relativistic particle:

$$W = \int_0^s F\,ds$$

$$F = \frac{dp}{dt} = \frac{d(\gamma mv)}{dt} \quad \Rightarrow \quad W = \int_0^s \frac{d(\gamma mv)}{dt}\,ds = \int_0^{mv} v\,d(\gamma mv) = \ldots = \gamma mc^2 - mc^2$$

If the particle started from rest the work goes into kinetic energy, so we have:

$$KE = \underbrace{\gamma mc^2}_{\text{total energy } E} - \underbrace{mc^2}_{\text{rest energy } E_0}$$

The conservation of mass energy

$$E = E_0 + KE$$
$$E_0 = mc^2$$

Energy and Momentum

How are relativistic energy and momentum related?

$$p = \gamma m v \Rightarrow p^2c^2 = \frac{m^2v^2c^2}{1-v^2/c^2}$$

$$E = \gamma mc^2 \Rightarrow E^2 = \frac{m^2c^4}{1-v^2/c^2}$$

$$\Rightarrow E^2 - p^2c^2 = \frac{m^2c^4(1-v^2/c^2)}{1-v^2/c^2} = \left(mc^2\right)^2$$

So the relation between energy and momentum is: $E^2 - p^2c^2 = E_0^2$

Note that $E^2 - p^2c^2$ is invariant; it is the same in all inertial frames.

Q: can a <u>massless</u> particle exist with energy and momentum?

Where Does This Leave Us?

- More to be added later