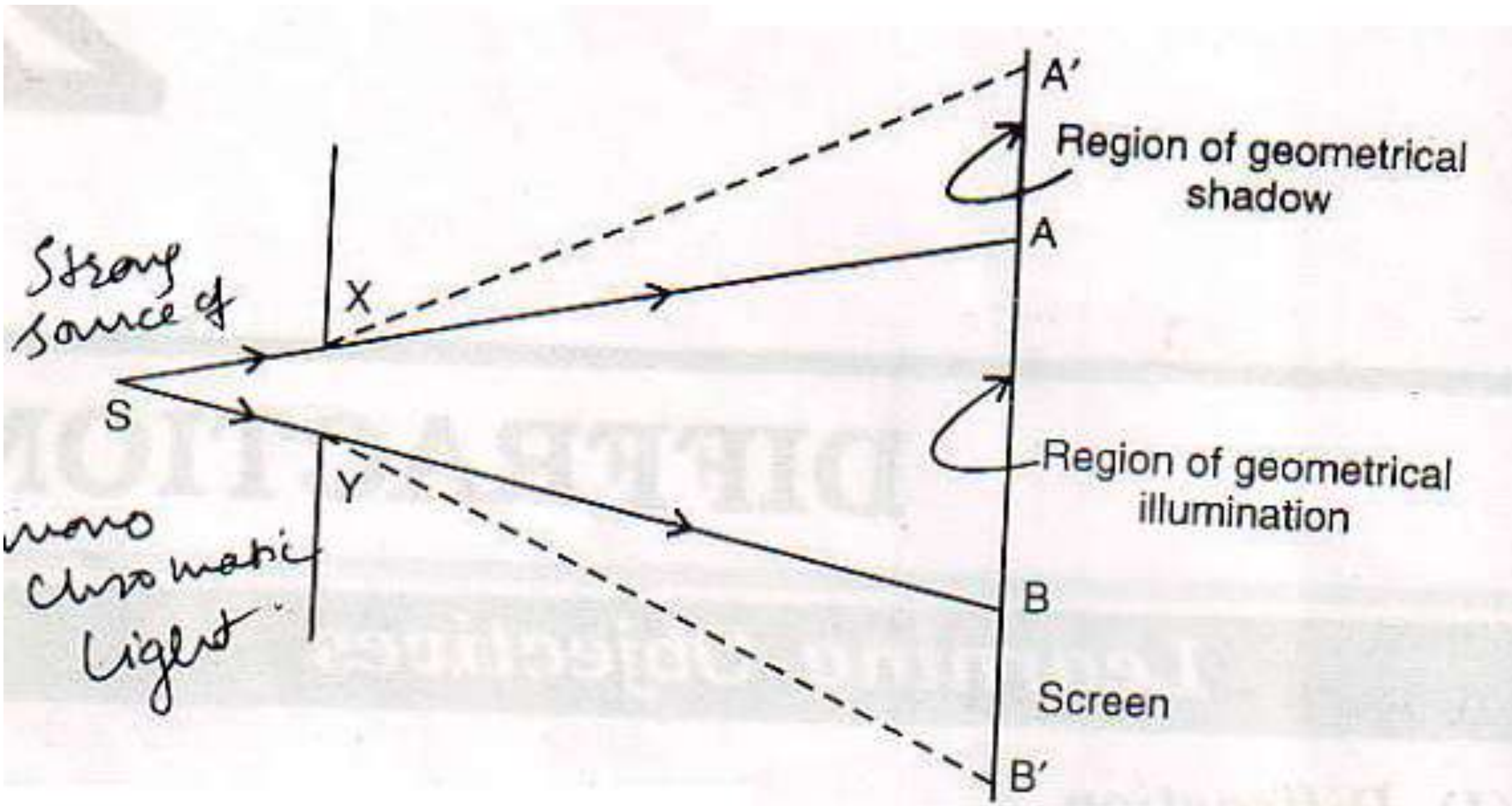
Diffraction

INTRODUCTION

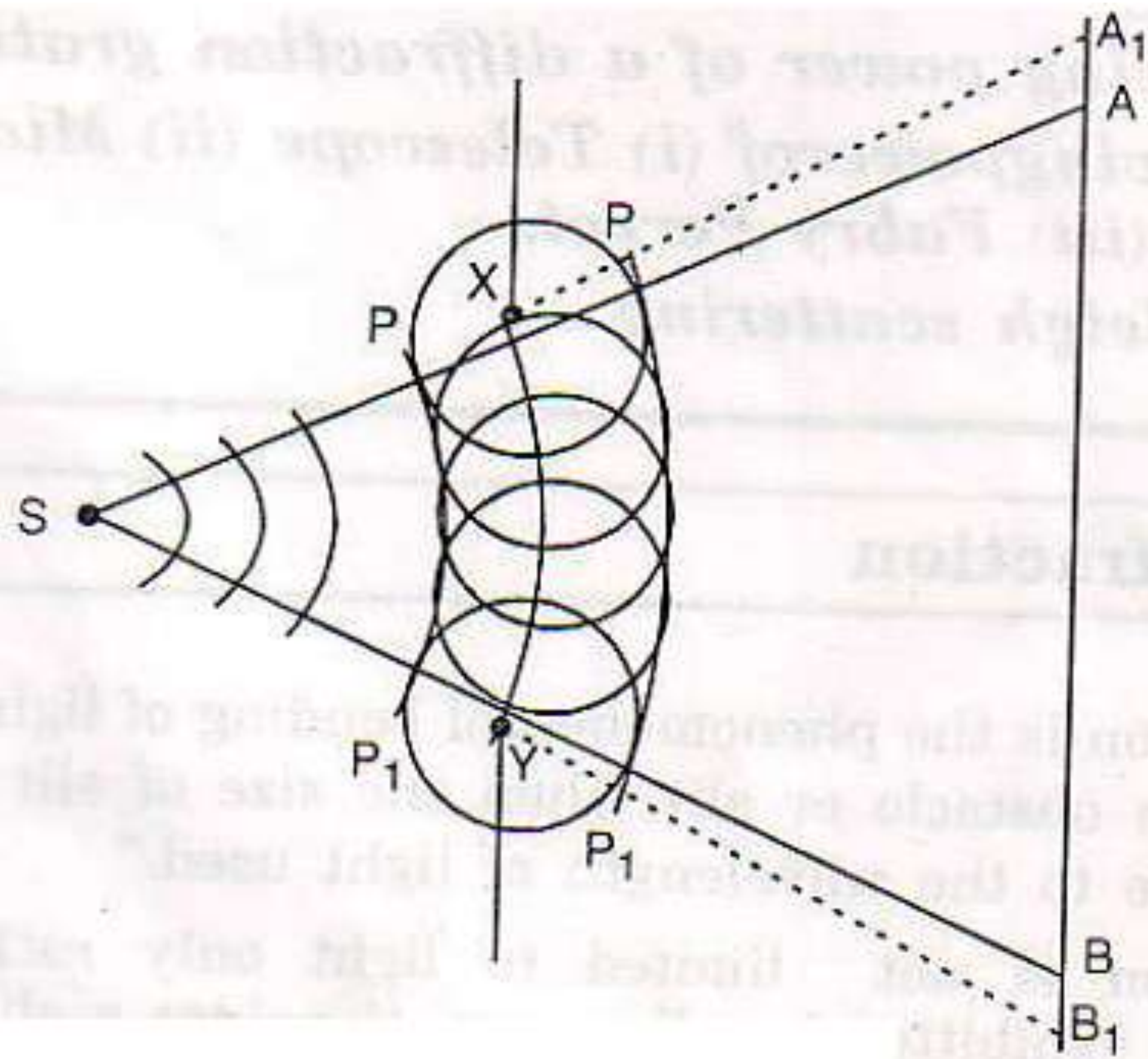
When waves encounter obstacles (openings), they bend round the edges of the obstacles, if the dimensions of the obstacles are comparable to the wavelength of the waves. This bending of light round the edges and spreading of light waves into the region of the geometrical shadow of the obstacle or aperture, is called diffraction of light.



A'
Region of geometrical shadow
A
Strong source of
X
S
Y
mono chromatic Light
Region of geometrical illumination
B
Screen
B'

Huygens Fresnel’s theory of diffraction

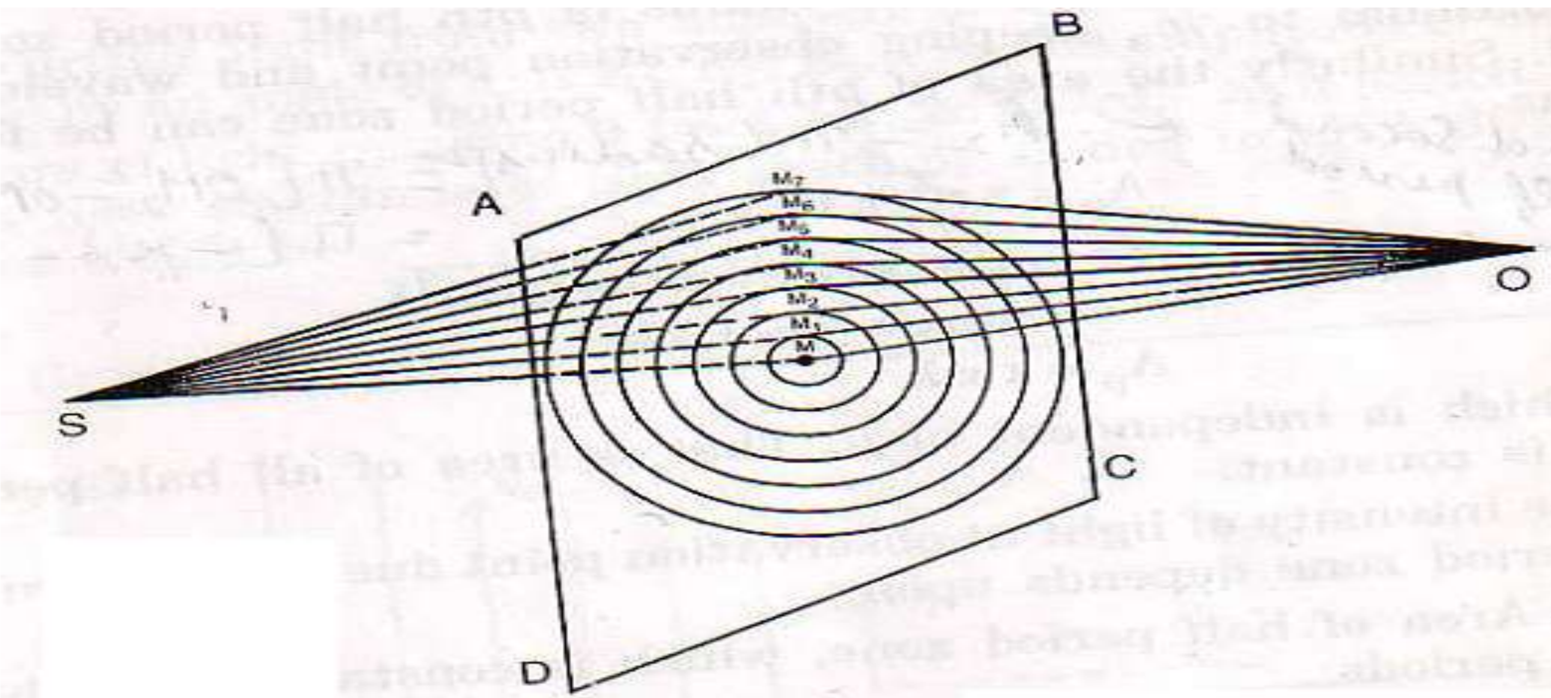
- According to Huygens principle , every point on the primary wave front acts as a source of a secondary wave front.
- Figure(in the next slide) shows a point source S of light from which spherical wave fronts are spreading out and are striking the slit XY.
- The parts of the secondary wave fronts originating from different points between X and Y also extend to the space above X and that below Y and may result in the illumination of the screen above A and below B.


A_1
A
P
X
P
S
P_1
Y
P_1
B
B_1

- Fresnel suggested that the secondary wavelets arrive in the region of geometrical shadows with varying phase relationship and amplitude.
- Thus secondary wave fronts originating from exposed part of primary wave front between X and Y explain the bending of the light round the corners of the slit. The intensity of light at a point on the screen is proportional to $(1+ \cos \theta)$.

FRESNEL'S HALF PRIOD ZONES

Let us consider a plane wave front ABCD coming from a distant monochromatic source of light(Fig. below)

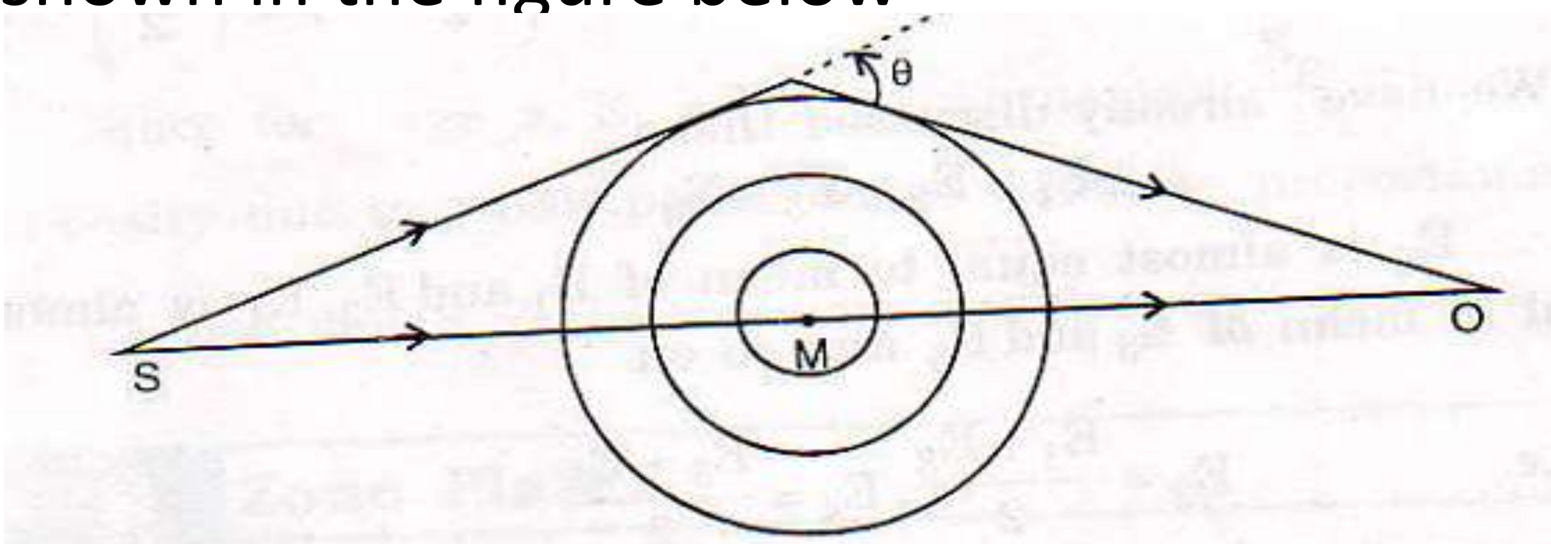


- Fresnel divided this plane into concentric circular regions referred to as Half period zones. Let light from source S forms primary wave front which are incident on this plane. Let O be any observation point. Let the line joining the source with observation point cut this plane at a point M.This point M is called “pole”. Let the observation point be at a distance x from this plane i.e. MO = x

- Now using M as centre, circles are drawn such that $M_1 O = x + \lambda/2$
- $M_2 O = x + 2\lambda/2$
- $M_3 O = x + 3\lambda/2$
- Where MM_1, MM_2, MM_3....are radii of various circles. Since light reaching at point O from adjacent circular strips is differing in path by $\lambda/2$ therefore such circular strips are called half period zones.

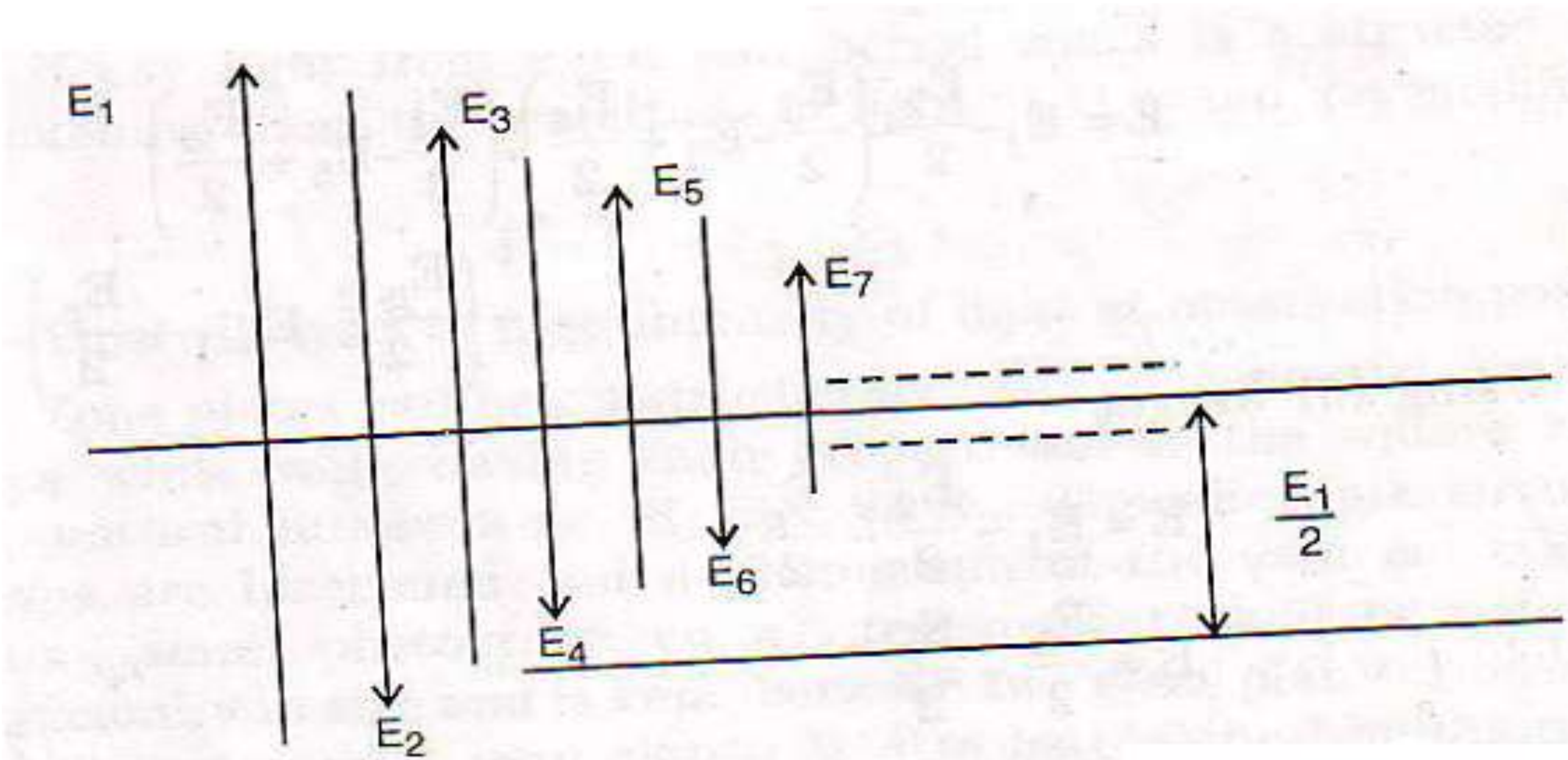
- The radius of p th half period zone is given by $r_p = (p \times \lambda)^{1/2}$
- Similarly the area of the p th half period zone is given by $A_p = \pi x\ \lambda$ which is independent of p.
- The intensity of light due to a particular half period zone depends upon
- 1)Area of the half period zone

- 2)Distance of half period zone from O
- 3)Obliquity i.e. as we move away from the pole, θ increases and hence intensity due to a particular half period zone decreases as shown in the figure below



- Further light from 2 adjacent half period zones differ in phase by an angle of π.
- If E_1 , E_2 so on…. be the amplitudes of electric vectors of light reaching O from 1st , 2nd …. So on half period zones then net amplitude of light reaching O due to all half period zones would be
- $E = E_1 - E_2 + E_3 - E_4 + E_5 - E_6 + \ldots\ldots + E_p$

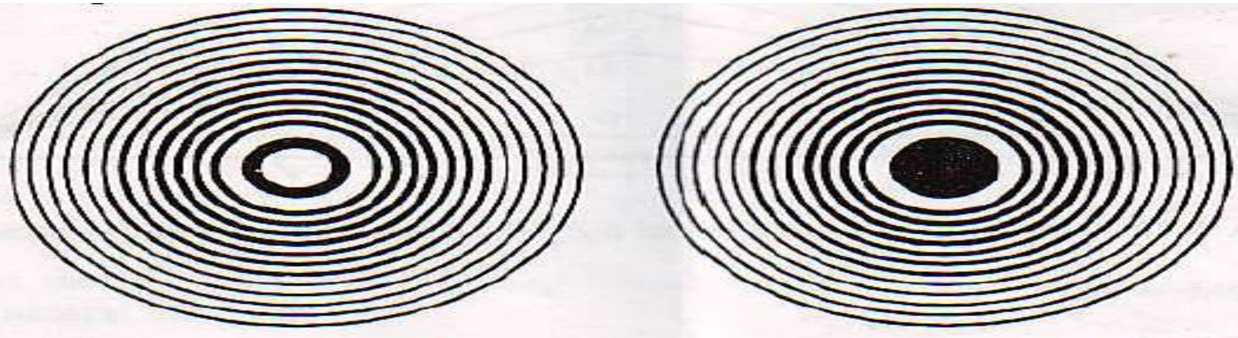
- Graphically this variation is shown in fig given below.



- Now total number of half period zones (i.e. p) may be even or odd.
- (1) p is odd.
- $E = E_1 – E_2 + E_3 – E_4 + E_5 – E_6 + ... + E_p$
- $= E_1 /2 + (E_1 /2 - E_2 + E_3 /2) + ... + (E_{p-2} /2 - E_{p-1} + E_p /2) + E_p /2$
- Now E_2 is almost equal to the mean of E_1 and E_3 and so on. Therefore all terms enclosed in brackets get vanished.

- i.e. $E = E_1/2 + E_p/2$
- (2) p is even.
- $E = E_1 - E_2 + E_3 - E_4 + E_5 - E_6 + \ldots\ldots - E_p$
-
- $= E_1 - E_2/2 - (E_2/2 - E_3 + E_4/2) - \ldots\ldots - (E_{p-2}/2 - E_{p-1} + E_p/2) - E_p/2$
- E_3 is almost equal to mean of E_2 and E_4 and so on. Therefore all terms enclosed in brackets get vanished.

- i.e. $E=E_1/2 - E_p/2$
- Thus for p half period zones total intensity is proportional to $(E_1/2 \pm E_p/2)^2$. Since for large p, $E_p << E_1$ so that total intensity due to p half period zones would be proportional to $E_1^2/4$.
- Zone Plate : A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. It can be designed so as to cut off light due to the even numbered zones or that due to the odd numbered zones.

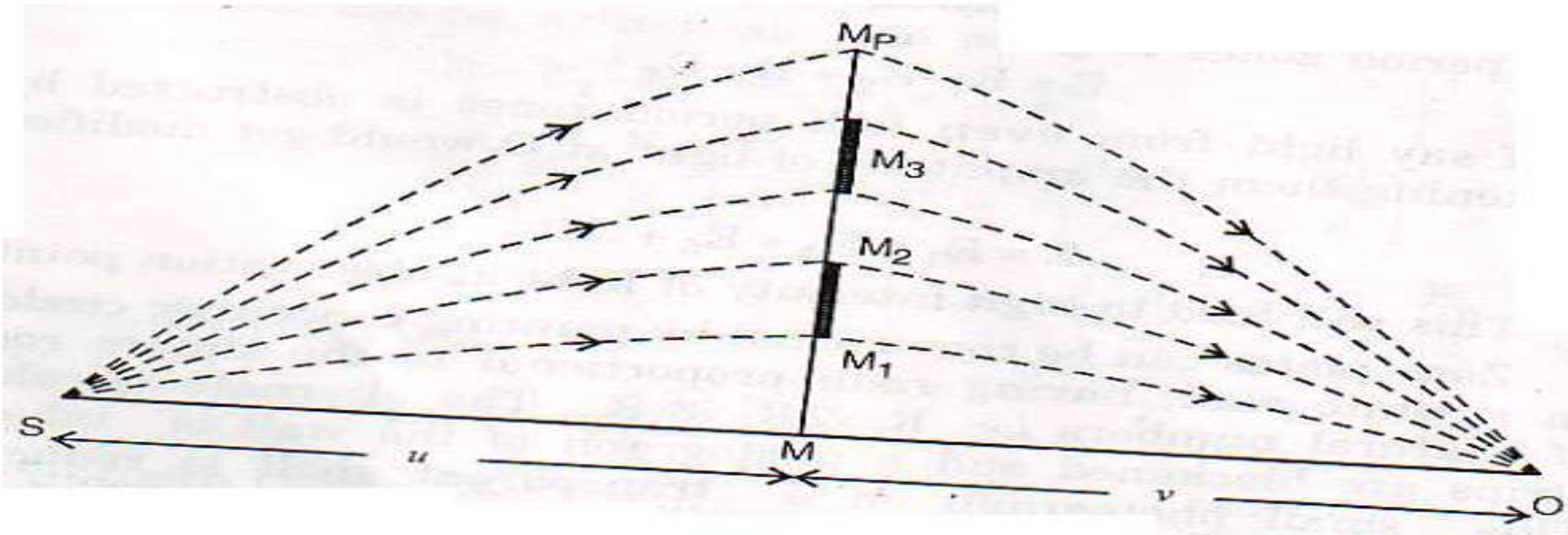


(a) A positive Z.P.   (b) A negative Z.P.

- Construction of the plate :To construct a zone plate , draw circles on a white card board piece with radii proportional to the square roots of the natural numbers. The odd numbered zones are covered with black ink . Take a photo of this pattern on a thin glass plate on a reduced scale. This forms the zone plate.

- Theory of the zone plate

- Consider a zone plate with alternate half period zones obstructing light. Let the source of light be at a distance of u from the zone plate. O be the observation point at a distance v from the zone plate. Let r_1, r_2.....r_p be radii of various half period zones. From the fig. shown in the next slide ,we have
- $SM+MO = u+v$



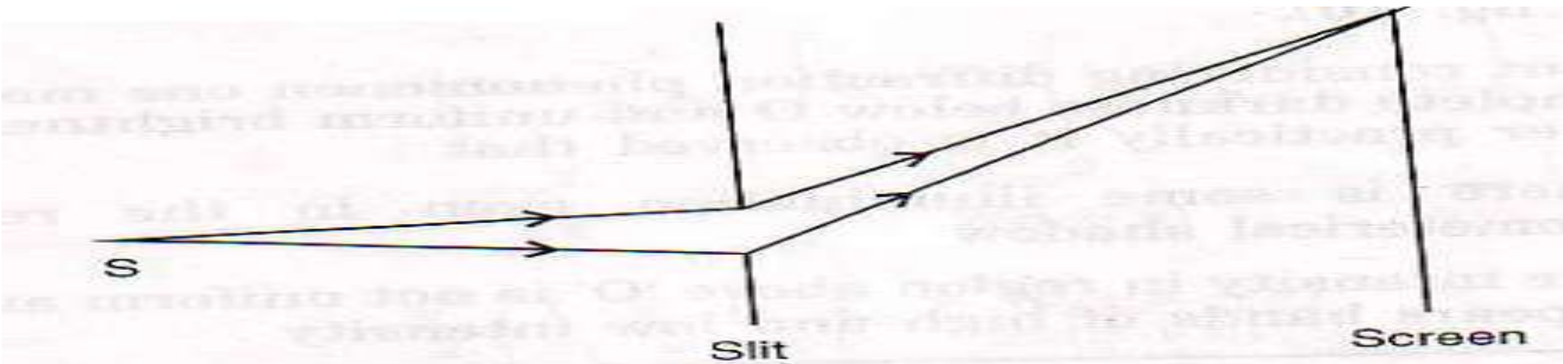
- $SM_1 + M_1O = u + v + \lambda/2$
- ………………………………….
- $SM_p + M_pO = u + v + p(\lambda/2)$……………….(1)

- $SM_p = u[1 + 1/2(r_p/u)^2]$
- $M_pO = v[1 + ½(r_p/v)^2]$
- Putting in equation no. 1 we finally get
- $1/u + 1/v = p \lambda/(r_p)^2$
- It is similar to lens' equation
- $1/u + 1/v = 1/ f_p$
- Where $f_p=(r_p)^2/p \lambda$

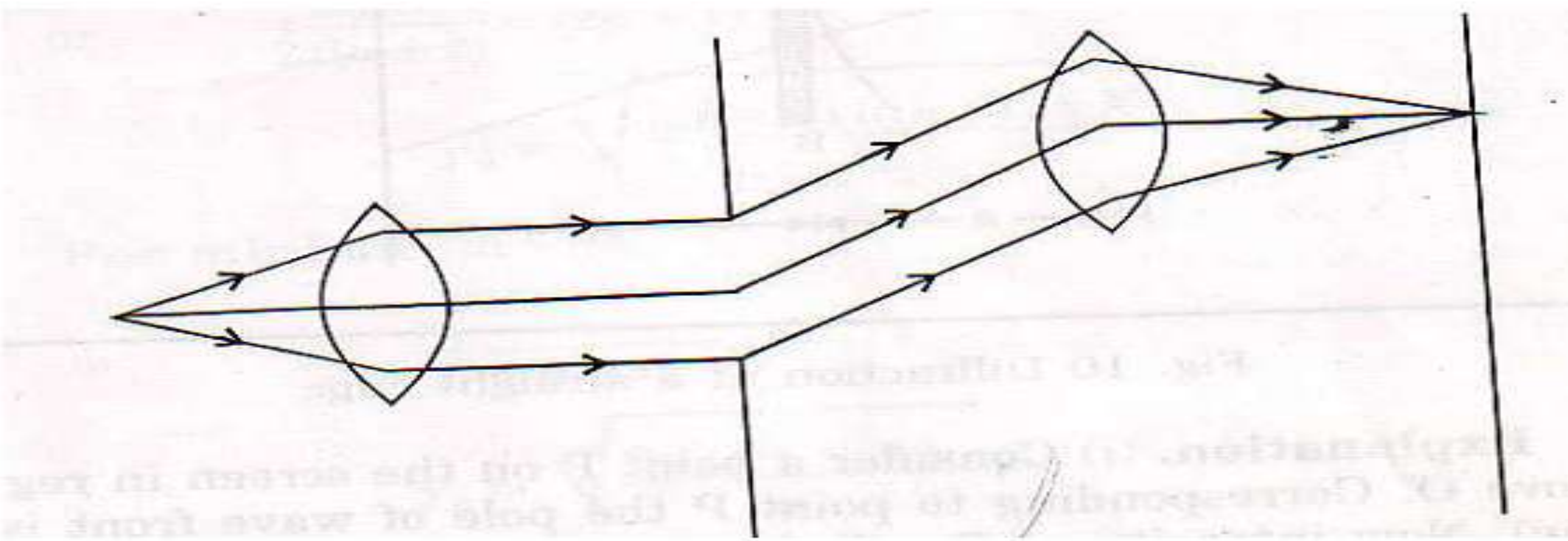
- Multiple Focii
- A zone plate has many focii. If the source is at infinity then value of v becomes equal to f_p. This focus is maximum bright. Practically zone plate and this focus are separated by many other points of high intensity at $f_p/3$, $f_p/5$, $f_p/7$….with intensity decreasing in the same order.

- Comparison of zone plate and convex lens
- 1.Focal length of zone plate is given as $f_p = r_p^2/p\lambda$, whereas focal length of a convex lens is given as $1/f = (\mu - 1)(1/R_1 + 1/R_2)$
- 2. Zone plate has many foci , whereas convex lens has just one focus.
- 3. Focal length of zone plate is inversely proportional to wavelength , whereas focal length of a convex lens is directly proportional to wavelength.

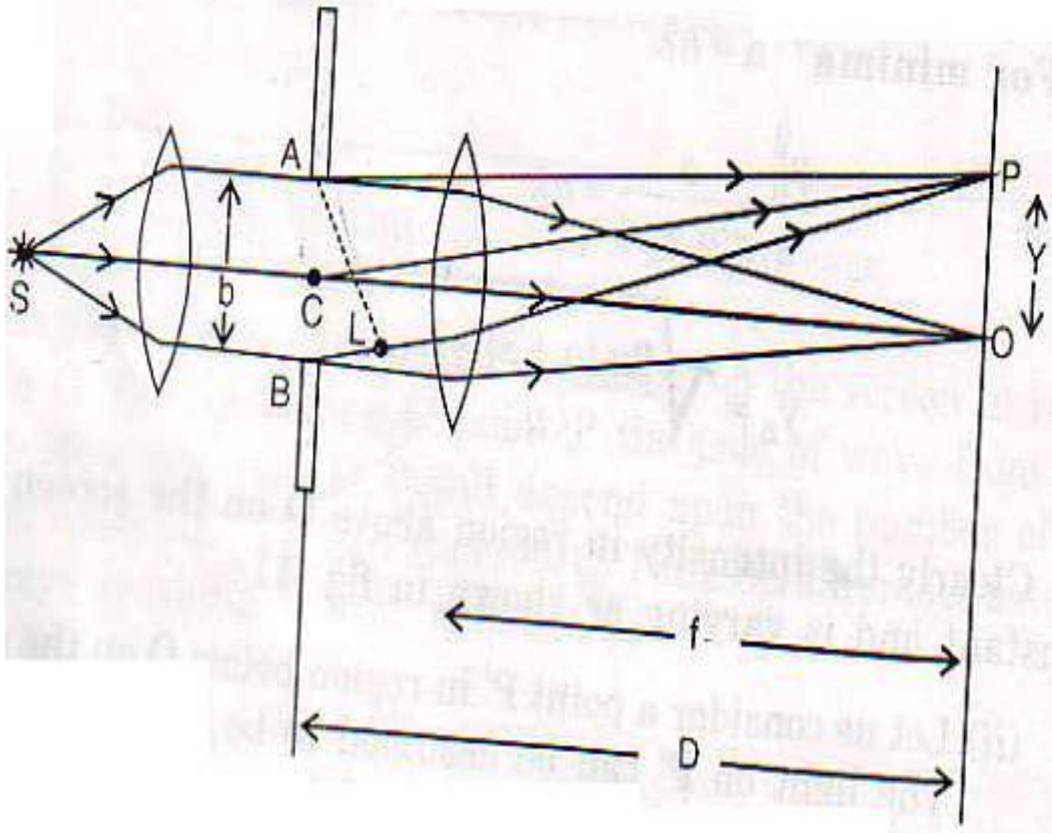
- Classification of diffraction
- There are two categories of diffraction.
- 1.Fresnel diffraction. In this type slit or obstacle is at finite distance from the source and obstacle. Fig. is shown below.



- Fraunhoffer diffraction
- In this type slit or obstacle are at infinite distance from the source or screen. Fig. is shown below.



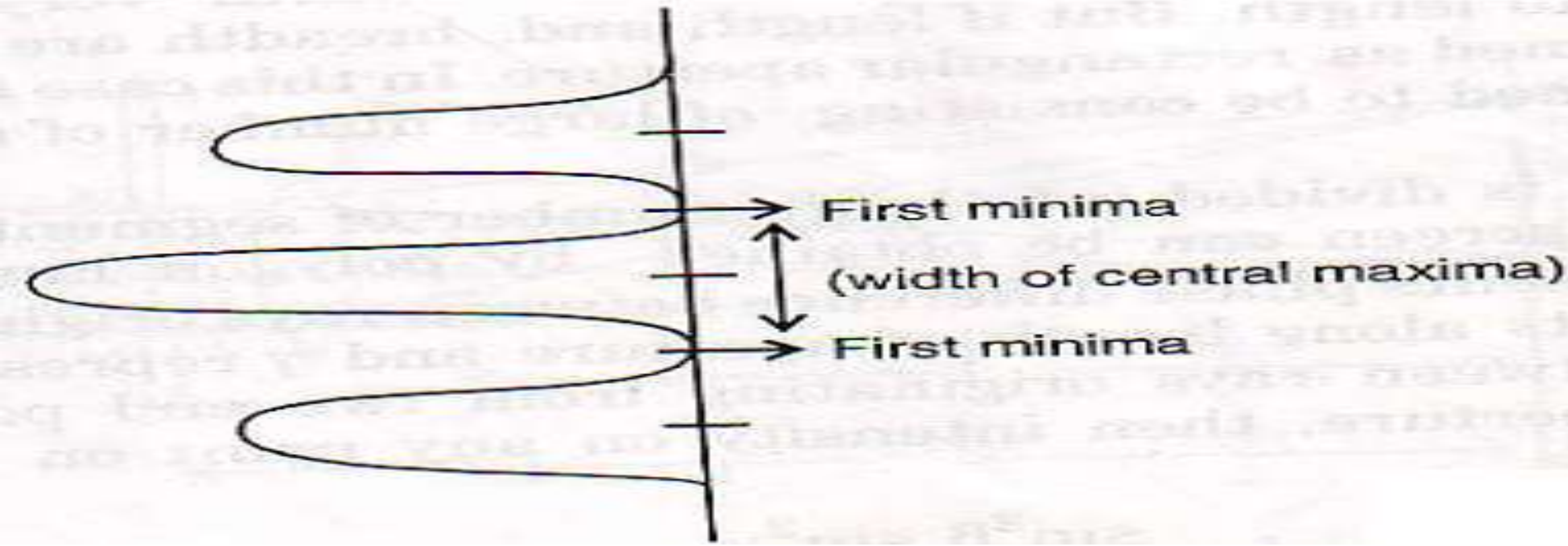
- Fraunhoffer Diffraction at single slit
- Consider a single slit AB on which a plane wave front is incident. The diffracted beam is brought to focus at some point P using a converging lens placed close to slit. This is shown in Fig. shown in the next slide. Let b be the width of the slit. The light reaching AB has all points in phase so point O is a maxima called central maxima. Let the diffracted beam make an angle θ with incident wave direction. Drop a perpendicular AL such that after AL all rays travel equal path.



S
A
B
C
L
b
P
Y
O
f
D

θ

- Path difference Δ = BL= b sin θ
- = n λ For minima
- = (2n + 1) $\lambda/2$ For maxima
- Width of central maxima. Width of central maxima is distance between first minimas on either side of centre of the screen.Fig. is as shown in the next slide.



- The following points can be noticed :

1) The width of the central maxima is inversely proportional to the slit width.
2) The width of the central maxima is directly proportional to wavelength.
3) The width of any secondary maxima is half of width of central maxima.

- Diffraction at Rectangular aperture
- A slit refers to an aperture with width very small in comparison to length. But if length and breadth are comparable then it is termed as rectangular aperture. Each slit is divided into large number of segments and total intensity on the screen can be found by polygon law of vector. Intensity at any point on the screen is found to be
- $I \propto \sin^2 \beta \sin^2 \gamma / \beta^2 \gamma^2$

- Where,
- $\beta = \pi b \sin\theta / \lambda$
- $\gamma = \pi l \sin\phi / \lambda$
- Diffraction pattern consists of central maximum with a number of maxima and minima spread alternately parallel to the length and breadth of the aperture.
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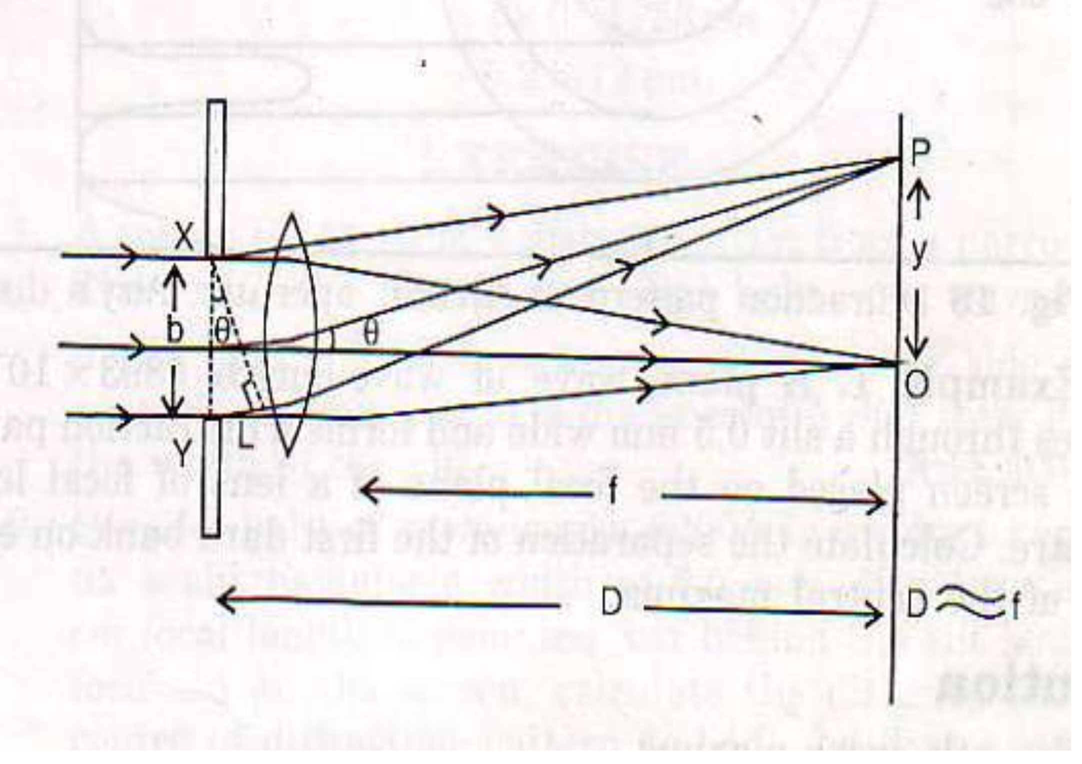
- Fraunhoffer diffraction at circular aperture
- Consider a plane wave front incident on a circular aperture .let b be the diameter of the aperture. Point O on the screen forms the central maxima. The path difference between the secondary waves reaching the screen at point P is given by
- $\Delta = b \sin\theta = n\lambda$ For minima
- $= (2n+1)\lambda/2$ For maxima

- Figure is shown below



- Fringe pattern consists of concentric bright and dark circles with bright circular disc at the centre having radius OP =y,is termed as Airy’s disc.
- The radius of Airy’s disc is given as
- $y=1.22\lambda f/b$(using precise measurements)