# INTERFERENCE OF LIGHT

INTRODUCTION : The term interference refers to a phenomenon involving superposition of waves leading to a modification of their intensity in the region of superposition. At certain points the intensity may become more than the sum of intensities of the individual waves and at certain other points it may tend zero. The former is known as constructive interference and the latter is called destructive interference . However the use of terms are misleading . Actually it causes redistribution of light rather than destruction or construction of light.

- Principle Of Superposition
- When two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

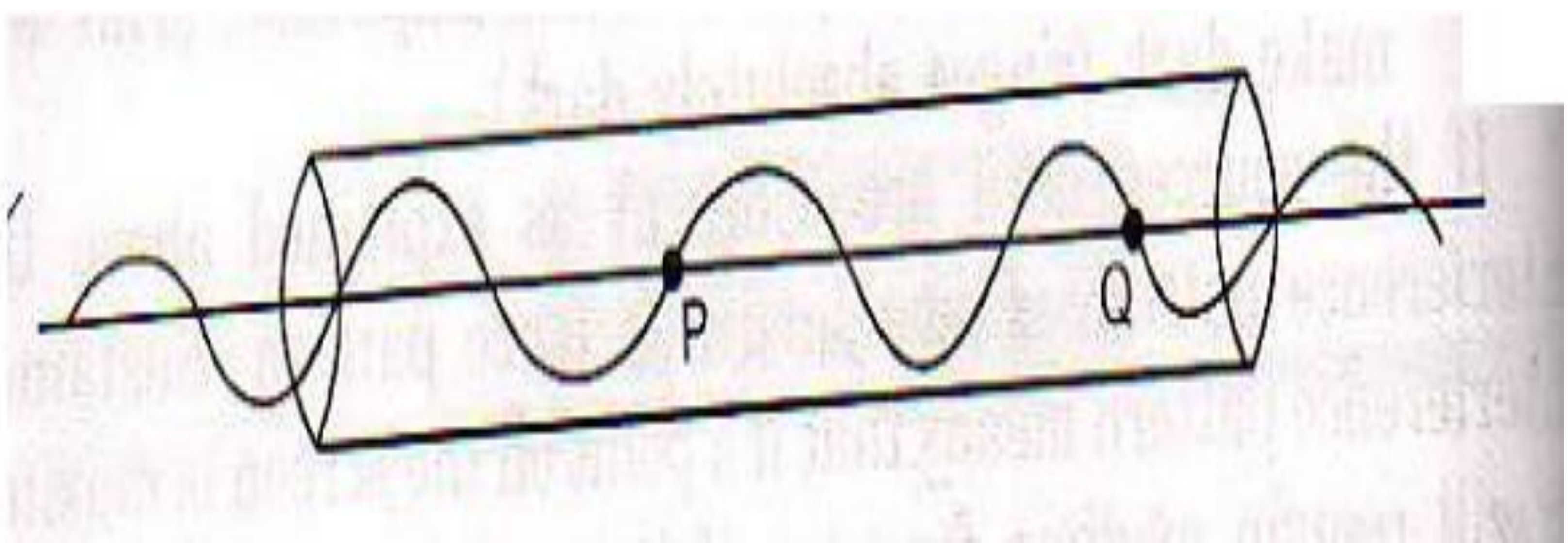
# COHERENCE

- It means the coordinated motion of several waves in a medium maintaining  a fixed and predicable phase relationship over a length of time . Two types of coherence

- 1. Temporal coherence
- 2. Spatial coherence

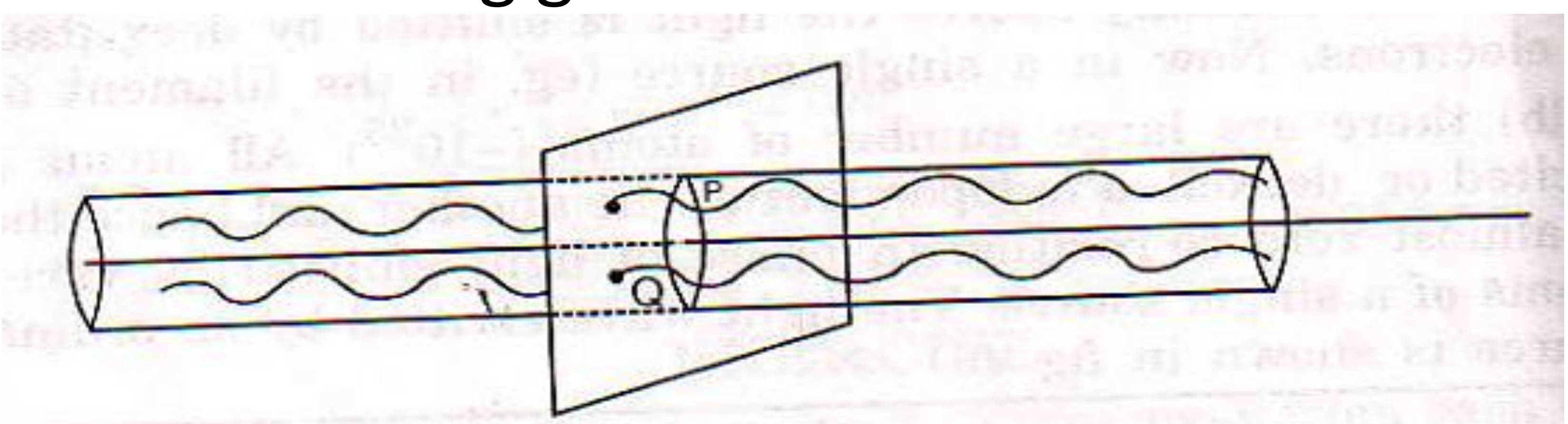
# TEMPORAL COHERENCE

- A beam of light is said to possess temporal coherence if phase difference of the waves crossing the two points lying along the direction of propagation of the beam is time independent. See fig below



# SPATIAL COHERENCE

- A beam of light is said to possess spatial coherence if the phase difference of the waves crossing the two points lying on a plane perpendicular to the direction of propagation of the beam is time independent. This is shown in Fig given below.



# MATHEMATICAL ANALYSIS OF TEMPORAL COHERENCE

- Light from real source consists of a series of harmonic wave trains which are characterized by coherent time or coherent length.

  **Coherent length ,** $l_0 = cT_0$ (Where $T_0$ is coherent time )

$$l_0 = c/\Delta\upsilon$$

$$= -\lambda^2/\Delta\lambda$$

**Line width** $= \Delta\lambda = -\lambda^2/l_0$

# Conditions for observing interference fringes

- Two interfering beams must satisfy the following conditions
- 1 .They should have either zero or constant phase difference.
- 2. They should be monochromatic and have same wave length.
- 3. They should have same state of polarization.
- 4. They should preferably have same amplitude.

- For obtaining sustained interference pattern following conditions should be satisfied

1. Two sources should be coherent in nature.
2. The light from two sources should have same state of polarization.

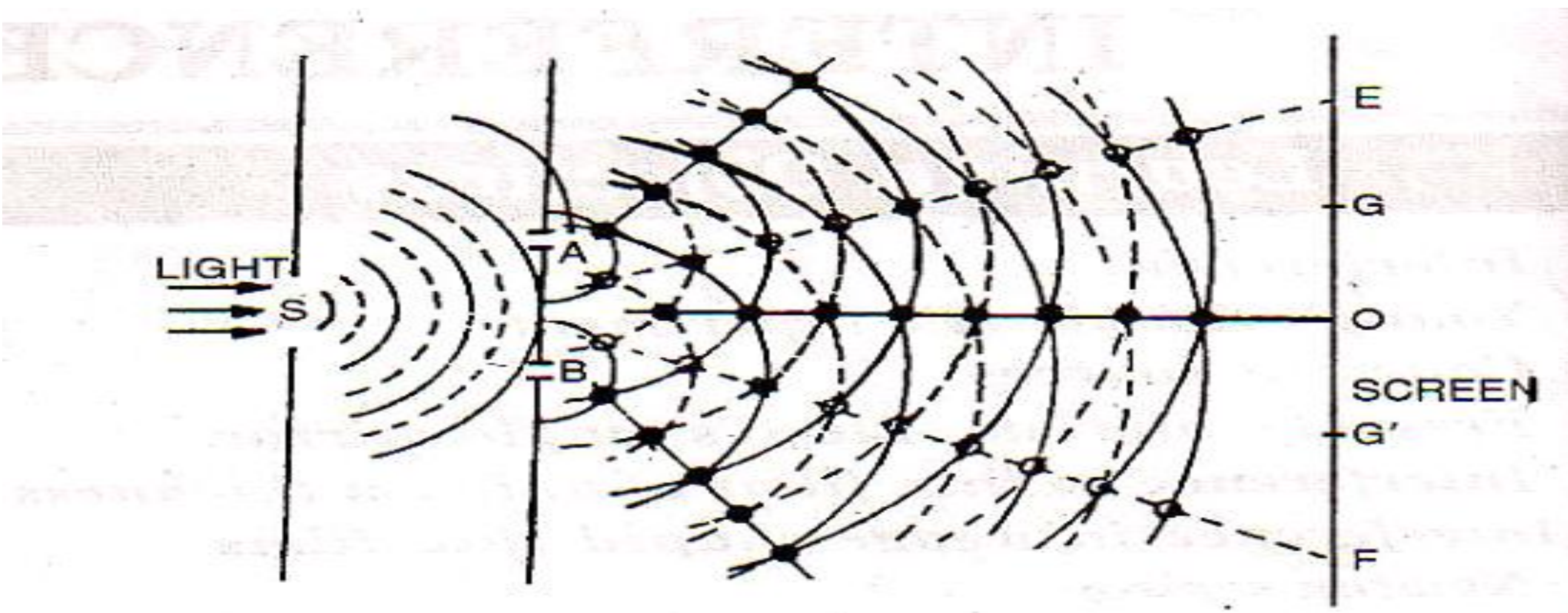
# Interference by wavefront division

- One of the methods consists in dividing a light wavefront, emerging from a narrow slit , by passing it through two slits closely spaced side by side. The two parts of the same wavefront travel through different paths and reunite on a screen to produce fringe pattern.
- Interference by amplitude division

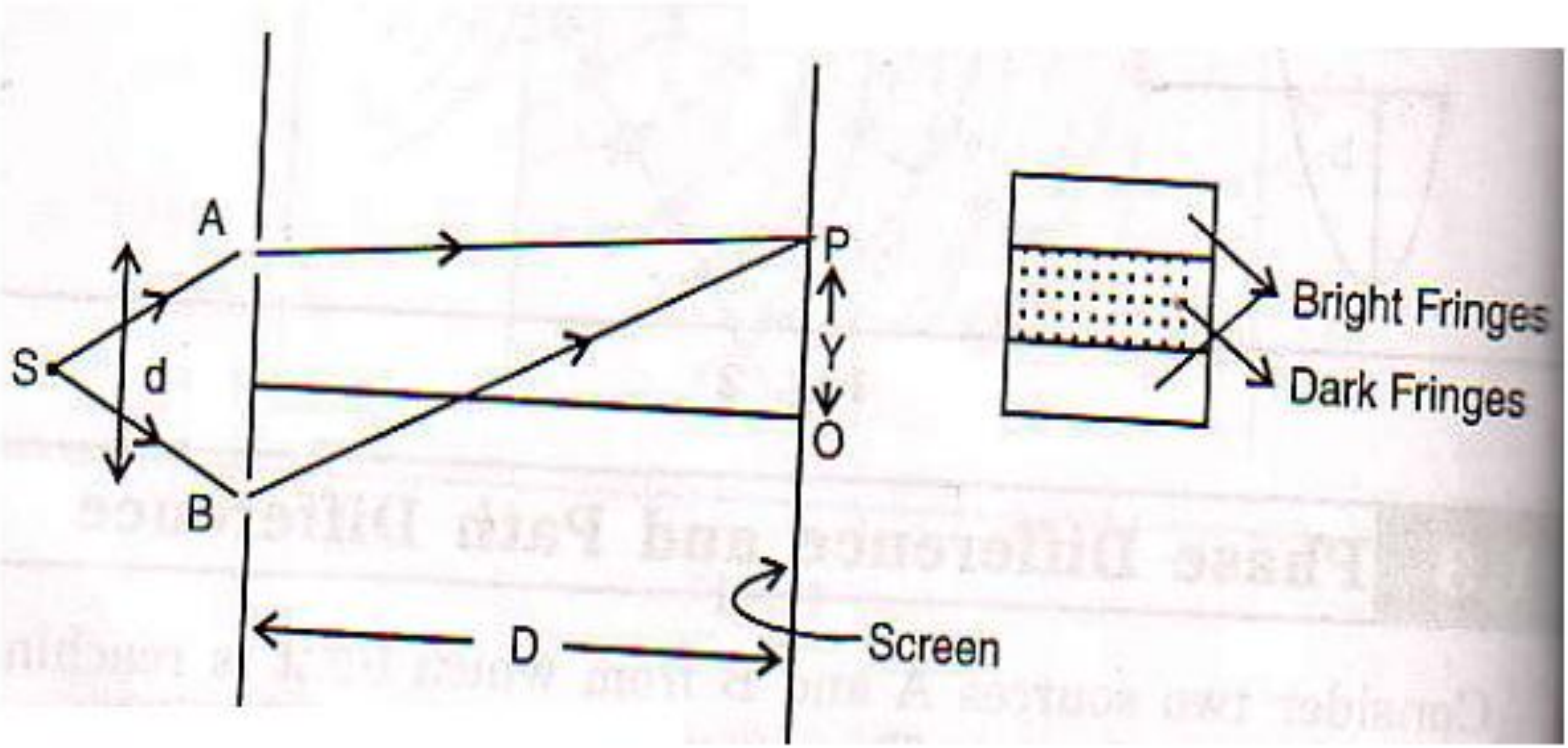
The amplitude of a light wave is divided into two parts, namely reflected and transmitted components, by partial reflection at a surface. The two parts travel through different paths and reunite to produce interference fringes.

# Young's double slit experiment

- Consider two pinholes A and B and light is allowed to fall on these pinholes from a source of light ' S' as shown in fig below. A and B are very close to each other and are placed symmetrically with respect to S.

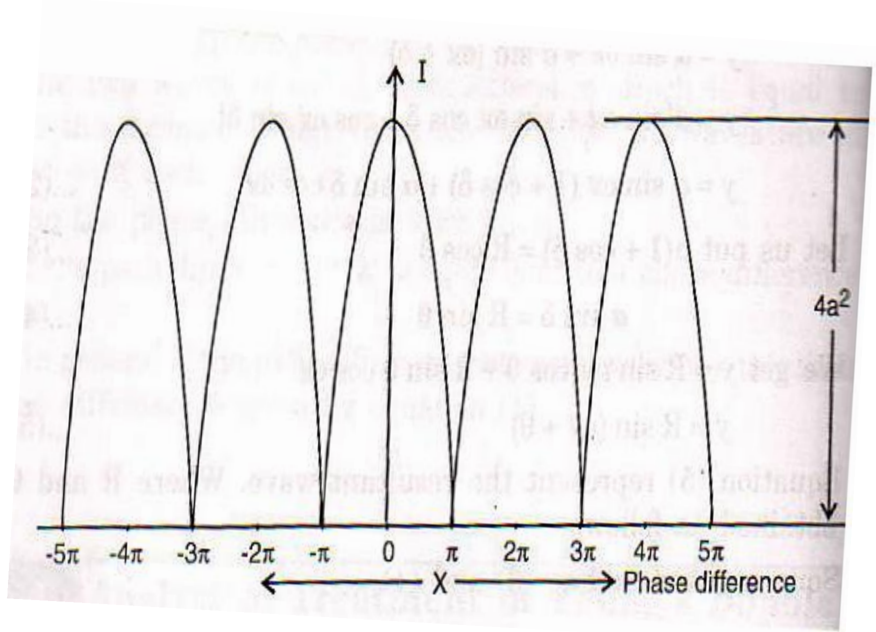


- When both slits are kept open we get bright and dark bands on the screen.
- Analytical Treatment of Young's Double Slit Experiment
- Consider source ' S' of monochromatic light. Let light from S falls on two slits A and B separated by distance 'd'. Let the distance of the screen from the slits be D as shown in fig in the next slide.


A
S
d
B
P
Y
O
D
Screen
Bright Fringes
Dark Fringes

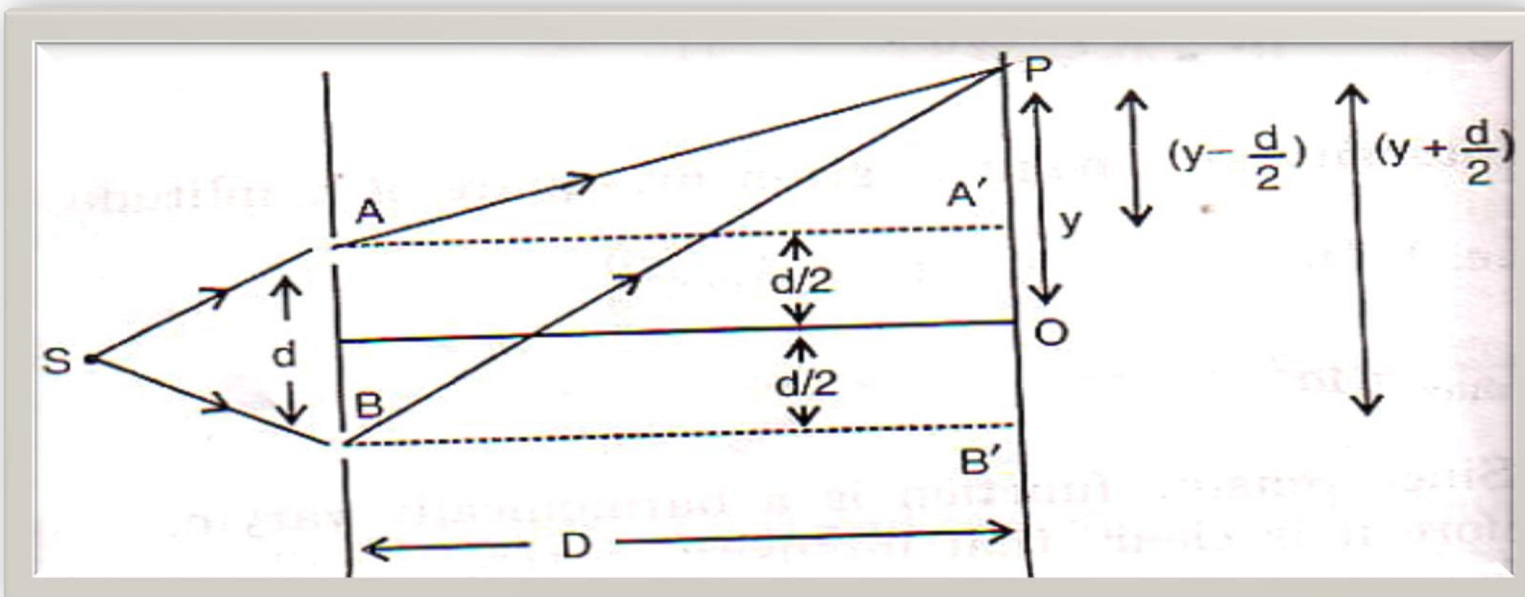
- The two waves can be represented as
- $Y_1 = a \sin \omega t$
- $Y_2 = a \sin (\omega t + \delta)$
- Then resultant wave is expressed as
- $Y = a \sin \omega t ( 1 + \cos \delta ) + a \sin \delta \cos \omega t$
- $= R \sin (\omega t + \theta)$
- Intensity at a point is given by
- $I = R^2 = 4a^2 \cos^2 \delta/2.$

- Since cosine function is a harmonically varying function therefore it is clear that intensity on screen will attain maxima and minima as shown in fig below



# Fringe Width in Double Slit Experiment

- Refer to fig below, the path difference between the rays reaching P from A and B is given as
- $\Delta = BP - AP$



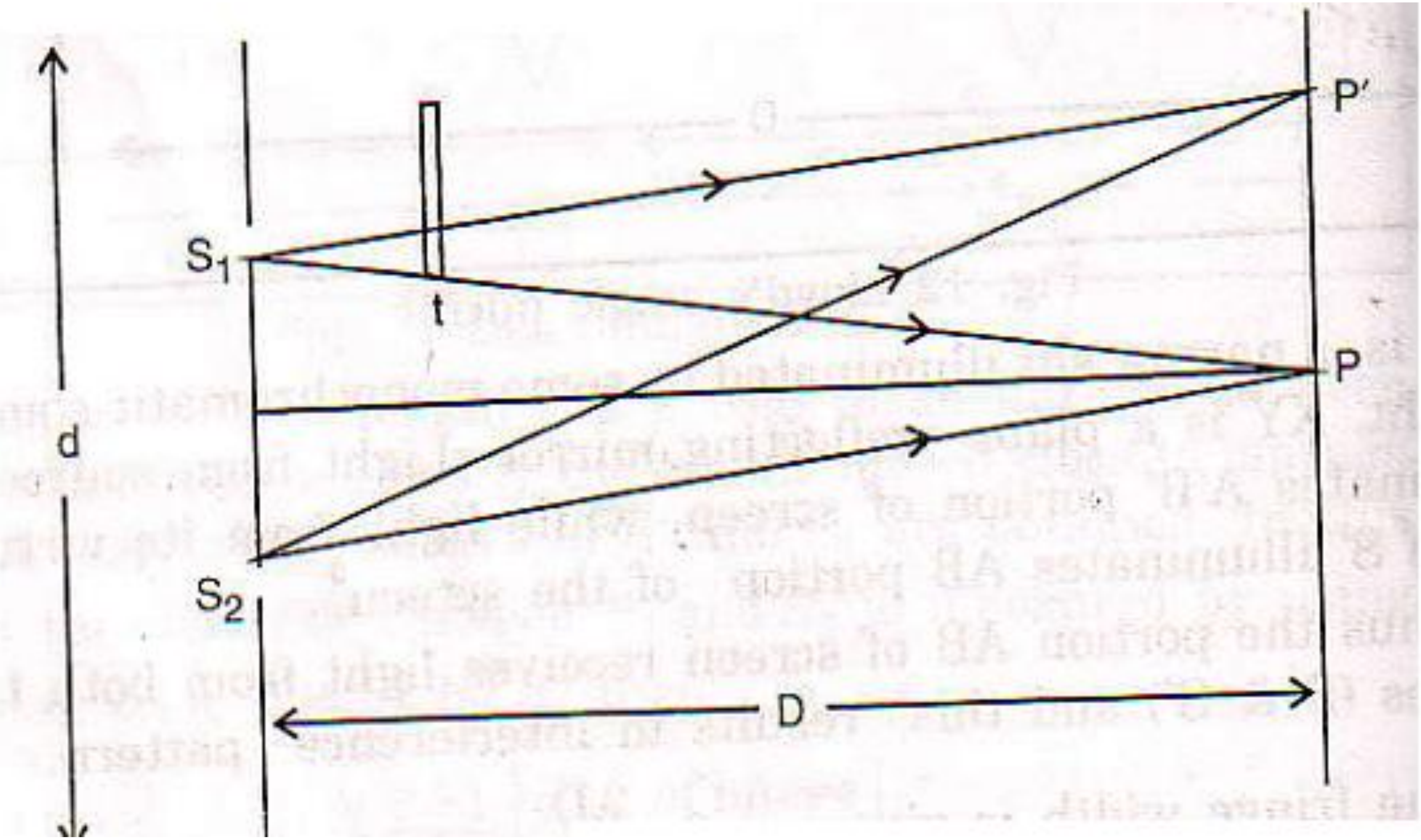
- Let the distance of point P from the centre of screen be y . Then
- $\Delta = y\, d/D$

For bright fringes , path difference is integral multiple of wavelength i.e. $\Delta = n\lambda$

  - $Y_n = n \lambda D / d$
  - Fringe width $= \beta = \lambda D / d$
  - Again distance between two consecutive dark fringes is given as
  - $\beta = \lambda D / d$ i.e. is interference pattern consists of dark and bright fringes of equal width with centre of screen forming a maxima.

# Determination of Thickness of sheet by interference

- We have seen in Young's double slit experiment that when light from two sources reach screen after travelling different paths they form an interference pattern on the screen. When a transparent sheet is placed in the path of one of the rays, it introduces addition path in that ray. Therefore fringe pattern shifts. This shift can be used to determine refractive index of the material

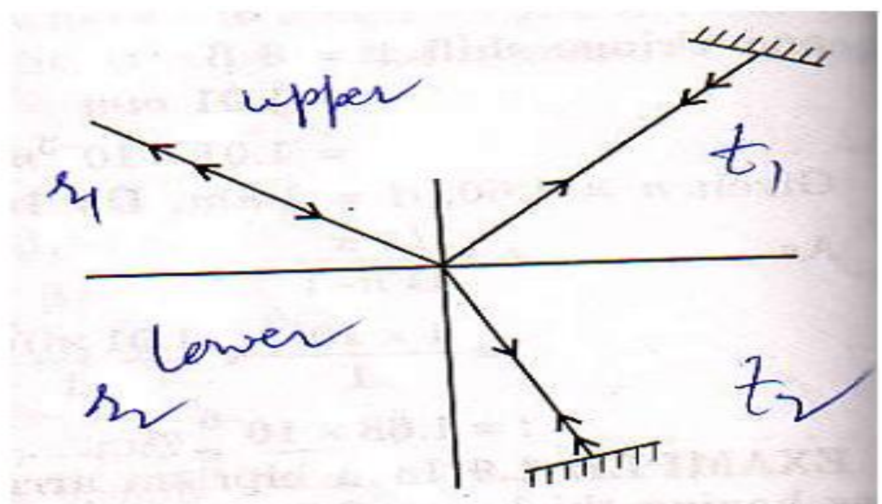

P′
S1
t
P
d
S2
D

- Let P be the position of the central maxima when there is no sheet in the path of any ray. Let P’ be the position of central maxima after a sheet of thickness ‘t’ is introduced in the path of one of the ray.
- Optical path from $S_1$ to P’ is given as
- $S_1$ P’-t+nt . On introduction of sheet in the path of one of the rays, central maximum shifts to point P’ which satisfies condition

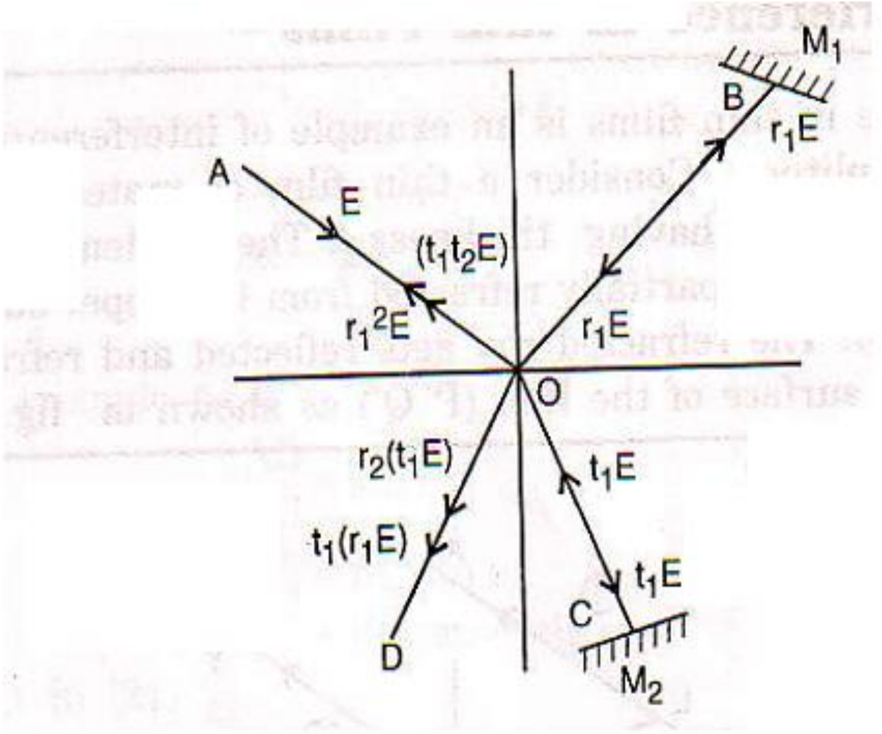
- $S_2P' = S_1P'$
- Therefore $S_2P' = S_1P' + (n-1)\,t$
- For bright fringe
- $$S_2P' - S_1P' = p\lambda$$
- If $PP' = x$ is the shift of central fringe then
- $p\,\lambda = xd/D = (n-1)t$
- $t = xd/(n-1)\,D = x\,\lambda\,/\,\beta\,(n-1)$ $\quad [\beta/\lambda = D/d]$

# Change of phase on Reflection

- According to the principle of reversibility of light ‘if the reflected and refracted light beams are made to retrace their paths by normal reflection then they join to recreate the original incident beam travelling in the opposite direction
- As show in the fig



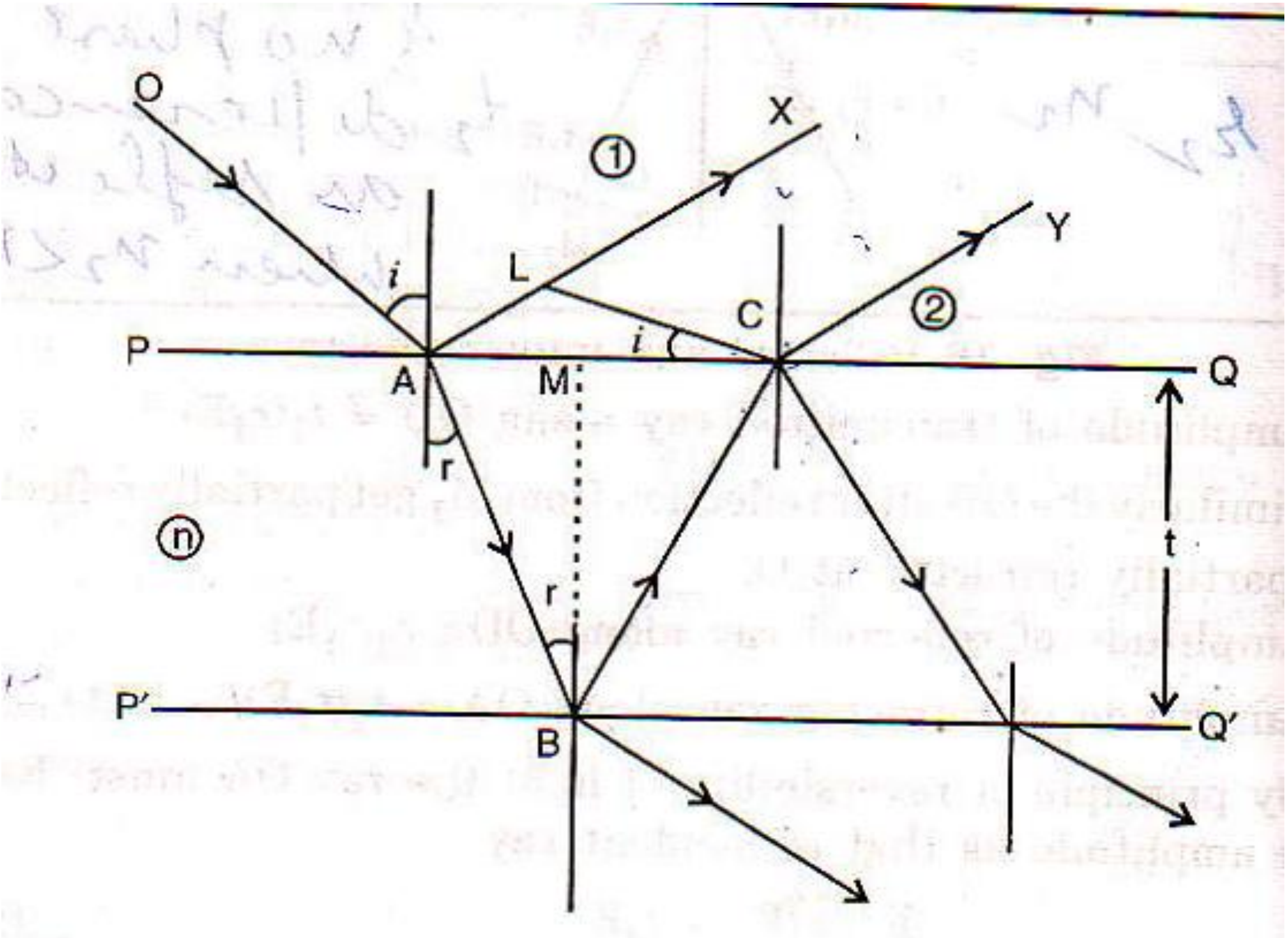
Let us consider that $r_1$ and $r_2$ be coefficient of reflection when light is incident on two sides of a surface XY. Let $t_1$ and $t_2$ be corresponding coefficients of transmission. Let E be the amplitude of electric vector of the incident ray. Then the amplitude of reflected ray is $r_1E$ and that of transmitted ray is $t_1E$,as shown in the fig



- By principle of reversibility of light the ray OA must have same amplitude as that of incident ray. Therefore $E = r_1^2 E + t_1 t_2 E$
- This is only possible if $r_1 t_1 E + t_1 r_2 E = 0$
- $r_1 = - r_2$

# Interference in thin Films

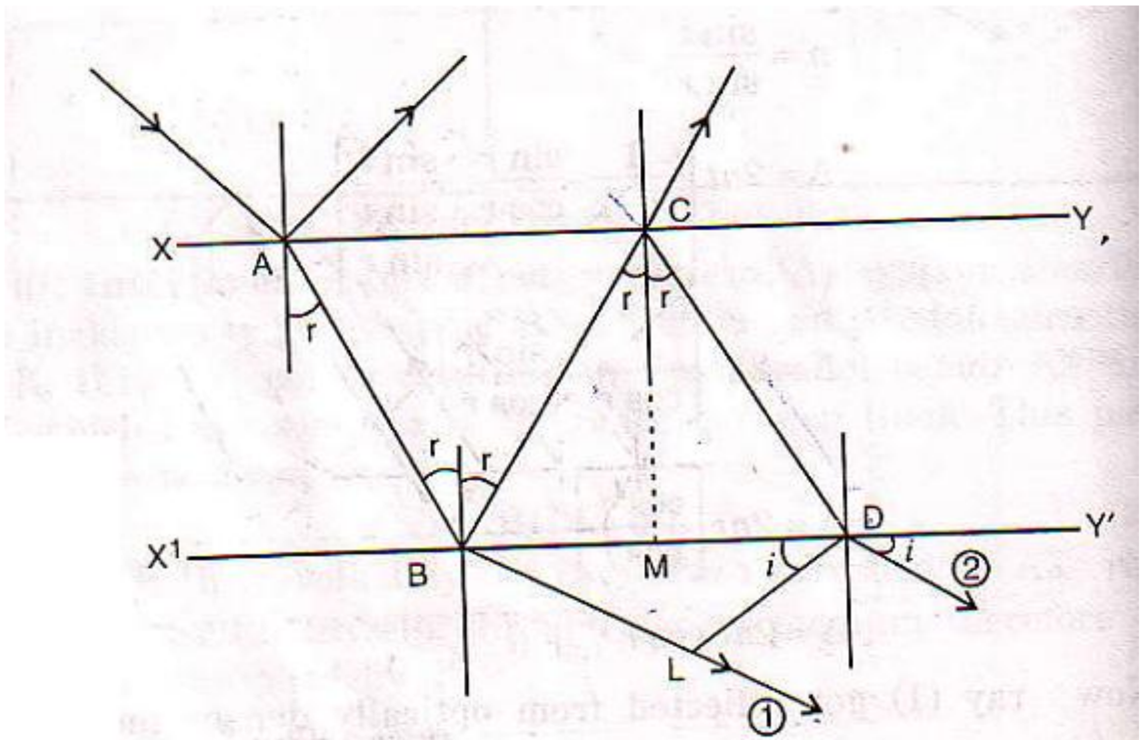
- Consider a thin film of material with refractive index n and having thickness t. The incident ray gets partially reflected and partially refracted from the upper surface of the film (PQ). The refracted ray gets reflected and refracted from the lower surface of the film (P’Q’) as shown in fig  shown in the next slide.


O
X
①
Y
L
i
C
②
i
P
A
M
Q
r
n
r
t
P′
B
Q′

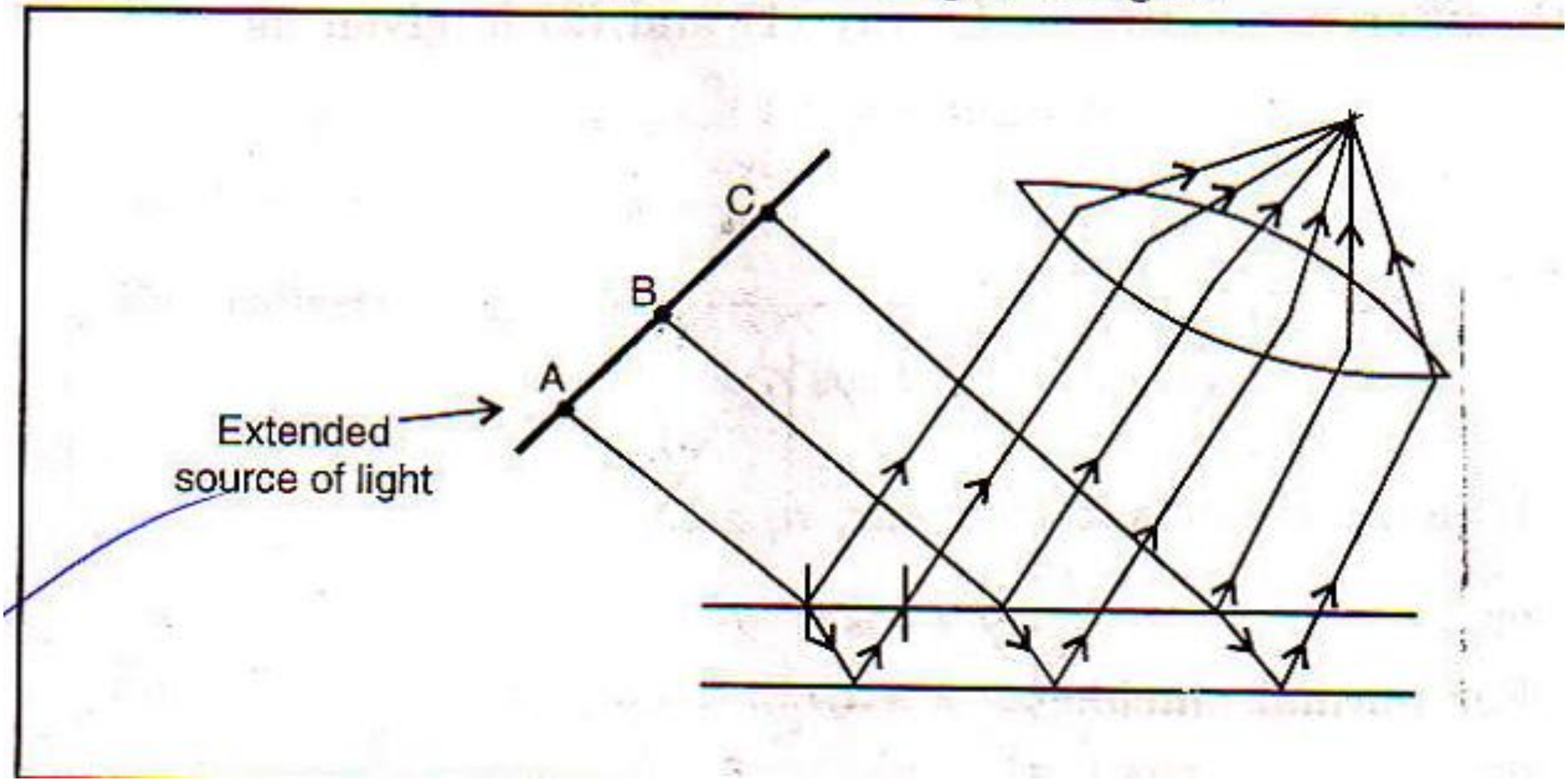
- (1) **Interference in reflected system**
- The incident ray suffers partial reflection and partial refraction at A, B and C points resulting in two parallel beams AX and AY which have path difference given as
- $\Delta$ = n(AB+BC)-AL
- =n( t/cos r +t/cos r) -2t tan r sin i
- = 2nt cos r

- Now ray (1) gets reflected from optically denser medium, therefore an additional path difference of $\lambda/2$ must be introduced. Hence real path difference is given as
- $\Delta' = \Delta - \lambda/2$
- $= 2nt \cos r - \lambda/2$
- For obtaining maxima $\Delta' = p\,\lambda$
- For obtaining minima $\Delta' = (2p - 1)\,\lambda/2$

# Interference in transmitted system

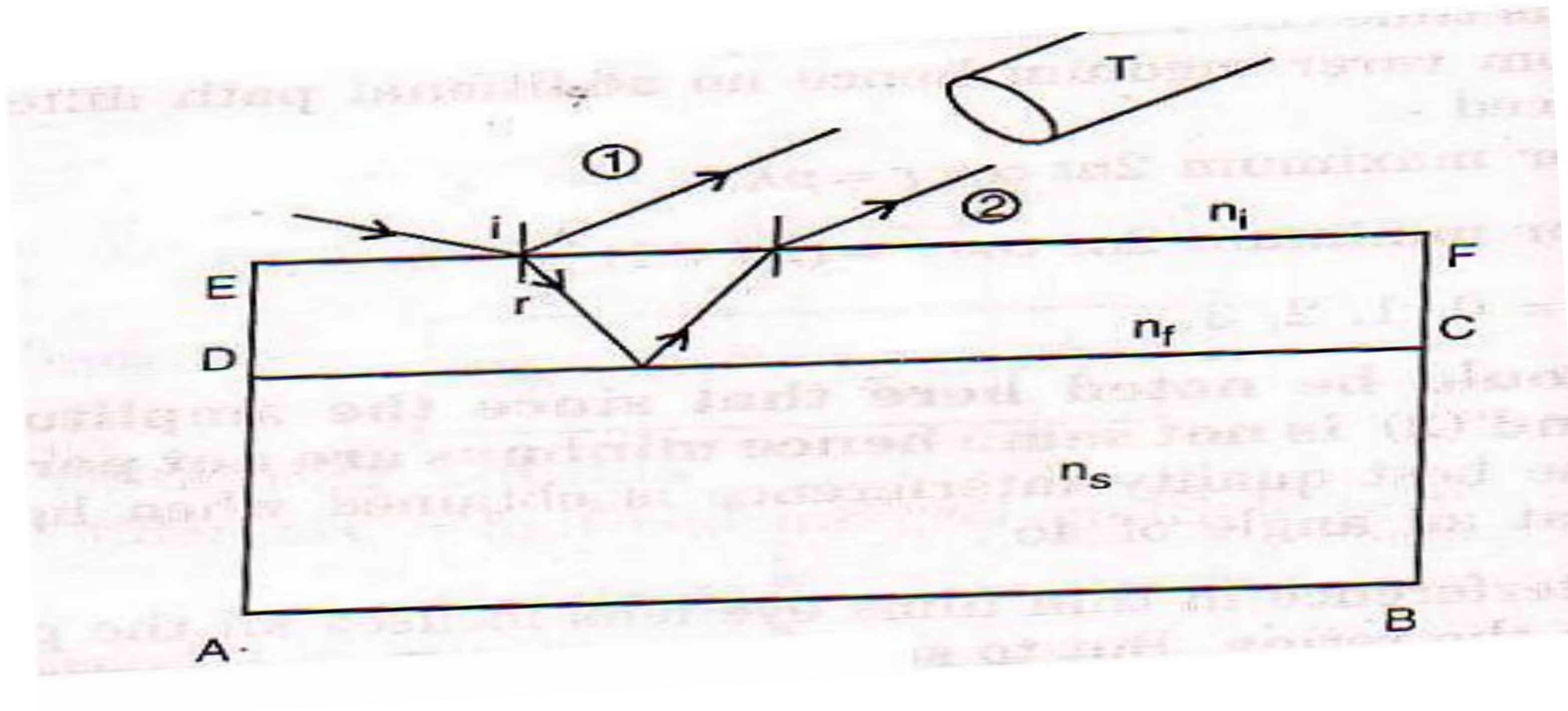


- The path difference in the ray (1) and (2) is given as
- $\Delta = n\ (BC + CD) - DL$
- $= 2\ n\ t \cos\ r$
- F or maximum $2nt \cos r = p\ \lambda$
- For minimum $2nt \cos r = (2p + 1)\ \lambda/2$
-


C
B
A
Extended
source of light

# Non Reflecting films

- A thin layer of a transparent medium is coated on lens or other optical devices which minimizes reflection from it. Such films are called non reflecting or anti reflecting films. Consider a glass slab ABCD. Let CDEF be thin film coated on this glass slab as shown in fig as shown in the next slide.




- Let $n_i$ ,$n_f$ ,ns be refractive indices of medium from which light is incident, film and slab. The path difference between the ray (1) and (2) is

- Given as
- $\Delta$ = 2nt cos r
- Minimum thickness of the film to be coated on the slab is, $t = \lambda_f/4$ and the two beams will interfere destructively for normal incidence.
- The coefficient of reflection of amplitude in terms of refractive indices is given as
- $r = (1-n)/(1+n)$
- It is found that if a material of refractive index 1.22 coated on glass slab with thickness $\lambda_f/4$ , it will serve as anti reflecting coating.
-

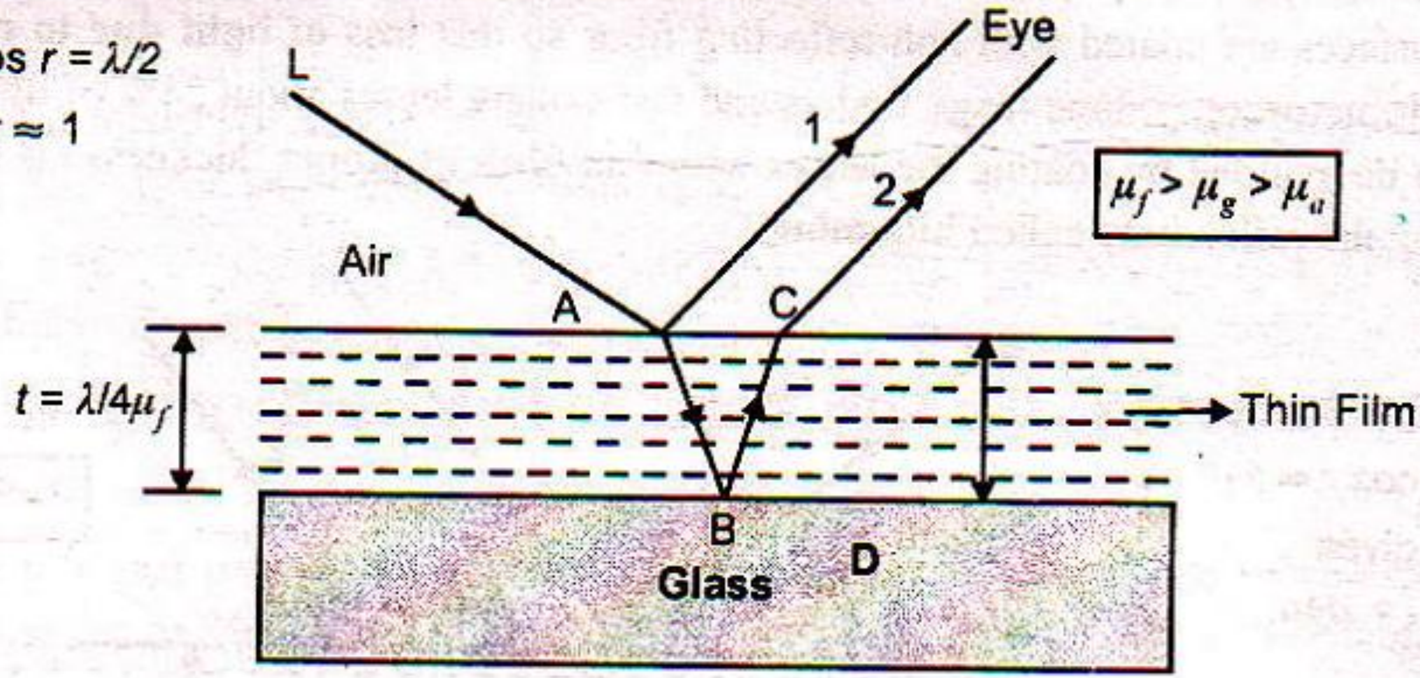
Let a ray of light LA be reflected from the dense film as ray (1)

If $2\mu_f \,.\, t \,.\, \cos r = \lambda/2$

then for $\cos r \approx 1$

we get

$t = \lambda/4\mu_f$

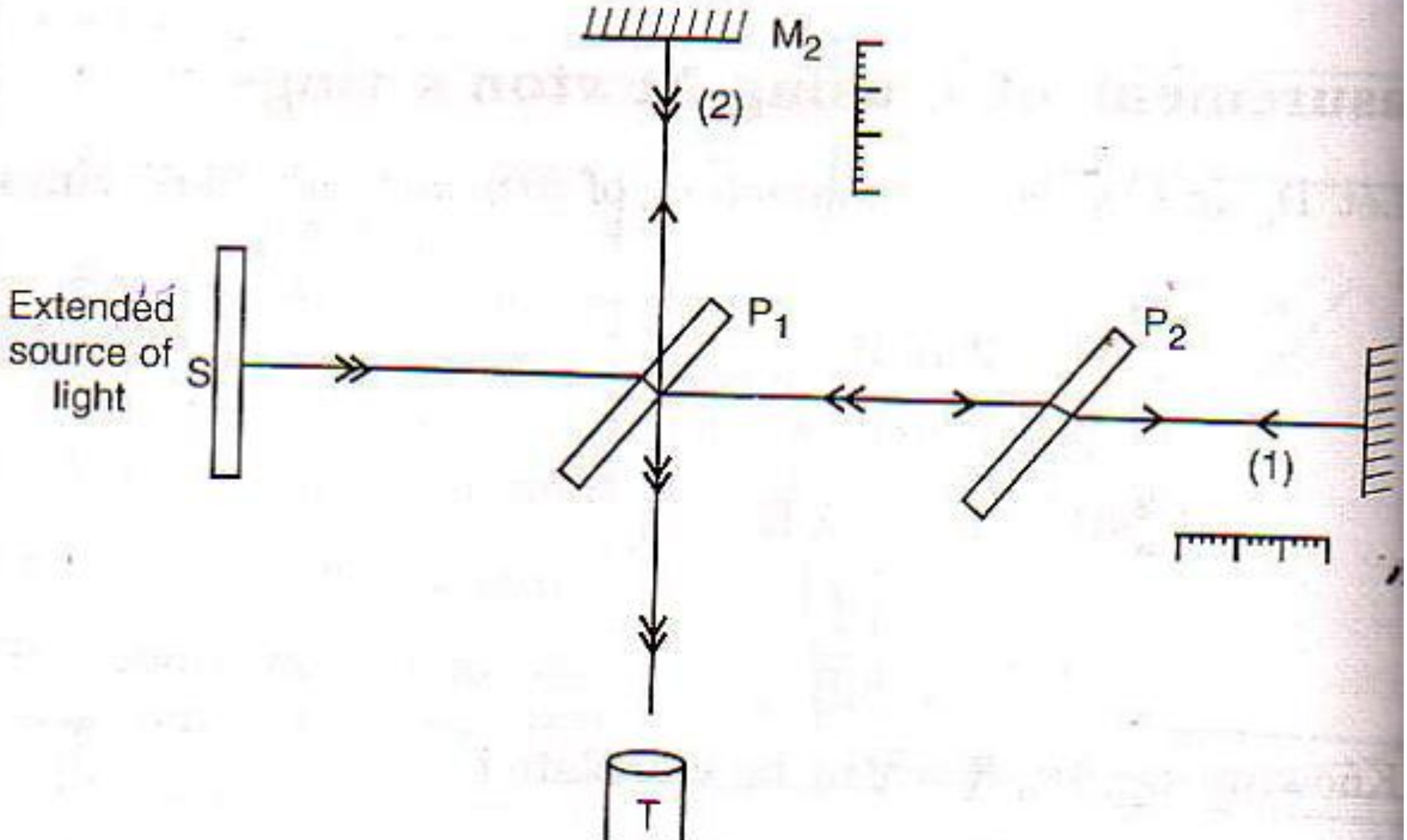


- The optical path between rays 1 and 2 is given by
- $\Delta = 2\ \mu_f\ t - \lambda/2$
- Where normal incidence of light is assumed. The condition for more reflection is
- $2\ \mu_f\ t - \lambda/2 = p\ \lambda$
- p = 0 gives the minimum thickness of the coating
- $\therefore \qquad t_{min} = \lambda / 4\ \mu_f$

# Michelson’s Interferometer

- PRINCIPLE
  In Michelson interferometer, a beam of light from an extended source is divided into two parts of equal intensities by partial reflection and refraction. These beams travel in two mutually perpendicular directions and come together after reflection from plane mirrors. The beams overlap on each other and produce interference fringes.

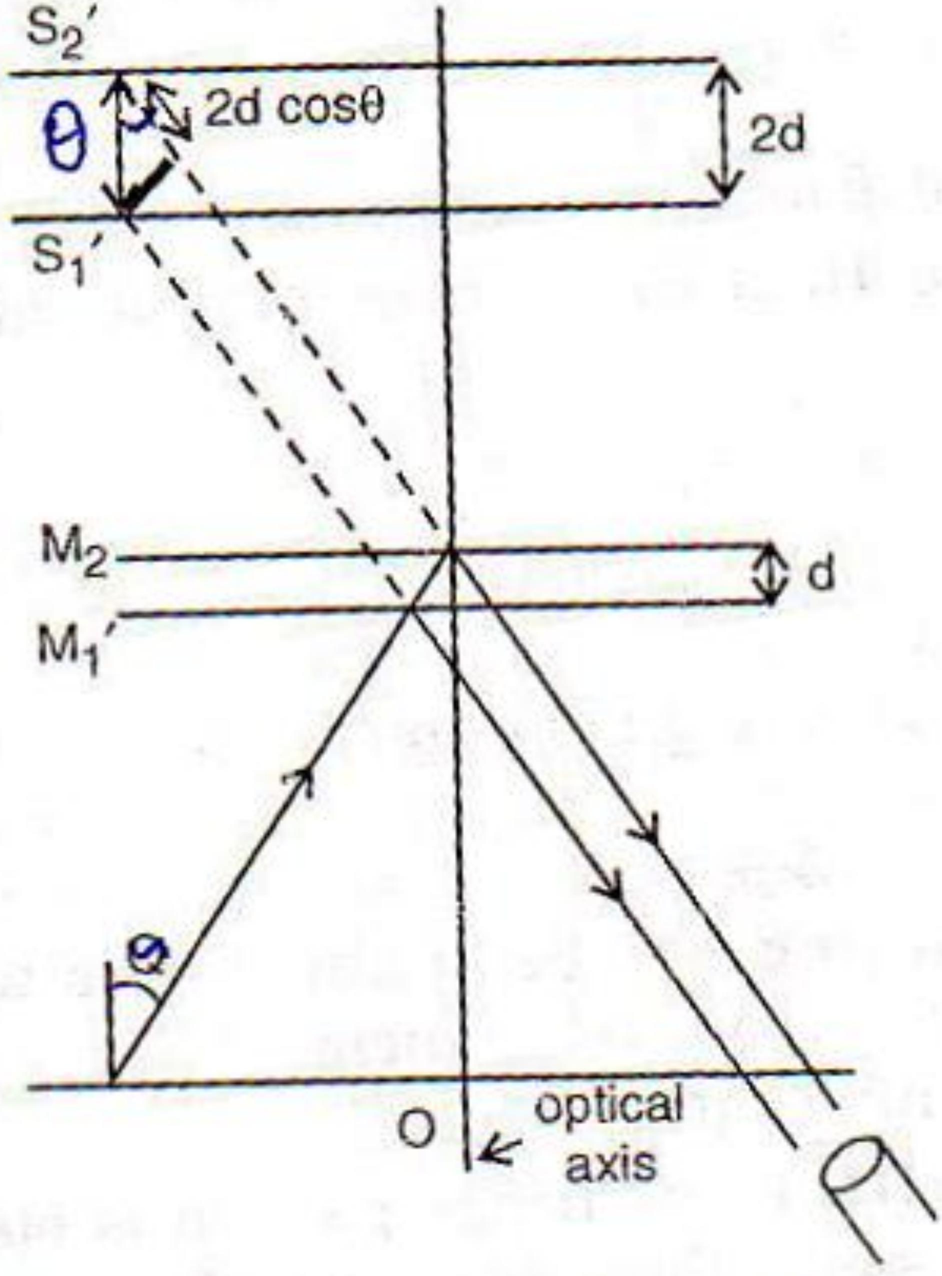
- WORKING
- The light from source S is incident on a glass plate $P_1$ which is partially reflecting and partially transmitting in nature. The beams after normal reflection from mirrors $M_1$ and $M_2$ interfere and fringe pattern is obtained. To make optical paths of the two rays ,(1) and (2) equal a similar glass plate $P_2$ is placed in the path of ray (1).The experiment set up is shown in the next slide.



M2
(2)
Extended
source of
light
S
P1
P2
(1)
T

THEORY

To simplify the analyses of the formation fringes, we consider an equivalent optical system as shown in the fig below.


S2′
θ
2d cosθ
2d
S1′
M2
d
M1′
θ
O
optical
axis

- Source S’ has been replaced b its images $S_1$’ & $S_2$’ in the two mirrors. If the separation between $M_1$’ & $M_2$ is d then separation between $S_1$’ & $S_2$’ will be 2d . Hence optical path difference between the reflected beams going to I will be $\Delta$=2 d cos θ
- As ray (1) suffers reflection from denser medium , an additional path difference of λ/2 is also introduced .
- So the modified path difference is given as
- $\Delta' = \Delta + \lambda/2$

- For maxima $\Delta' = p\lambda$
- For minima $\Delta' = (2p+1)\lambda/2$
- The interference pattern consists of circular fringes with their centre on the optic axis. The fringes are localized at infinity and can be seen by the eye or a telescope focussed t infinity.
- CASE 1.
- If d= 0 then $\Delta' = \lambda/2$ . That is central fringe will be dark.

- CASE 2.
- If the mirrors are moved apart such that $d = \lambda/4$ for $\theta = 0$
- We get $\Delta' = \lambda$ , Central fringe will be bright.
- CASE 3.
- If fringe pattern is observed for some oblique direction say $\theta$ then $2d \cos\theta = p\lambda$ . For a particular fringe say $p^{th}$ order fringe ,
- $2d \cos\theta$ = constant
- Or $\cos\theta \propto 1/d$, so as d increases ,the fringes appear to be expanding radially outward from the centre.

# Applications of Michelson's Interferometer

- (a) **Measurement of wavelength of source**
- If $\theta = 0$ (i.e. for normal incidence)
- $2d = p\lambda$
- or
- $\lambda = 2d/p$
- Now , one mirror is moved parallel to itself and the number of fringes crossing the field are continued.

- Let $d1$ be the distance between the two mirrors [in equivalent arrangement] for pth order dark ring and $d2$ is the distance for (p+m)th order dark ring then $2d1=p\ \lambda$
- $2d2=(p+m)\ \lambda$
- $\lambda= 2(d2-d1)/m=2\Delta d/m$

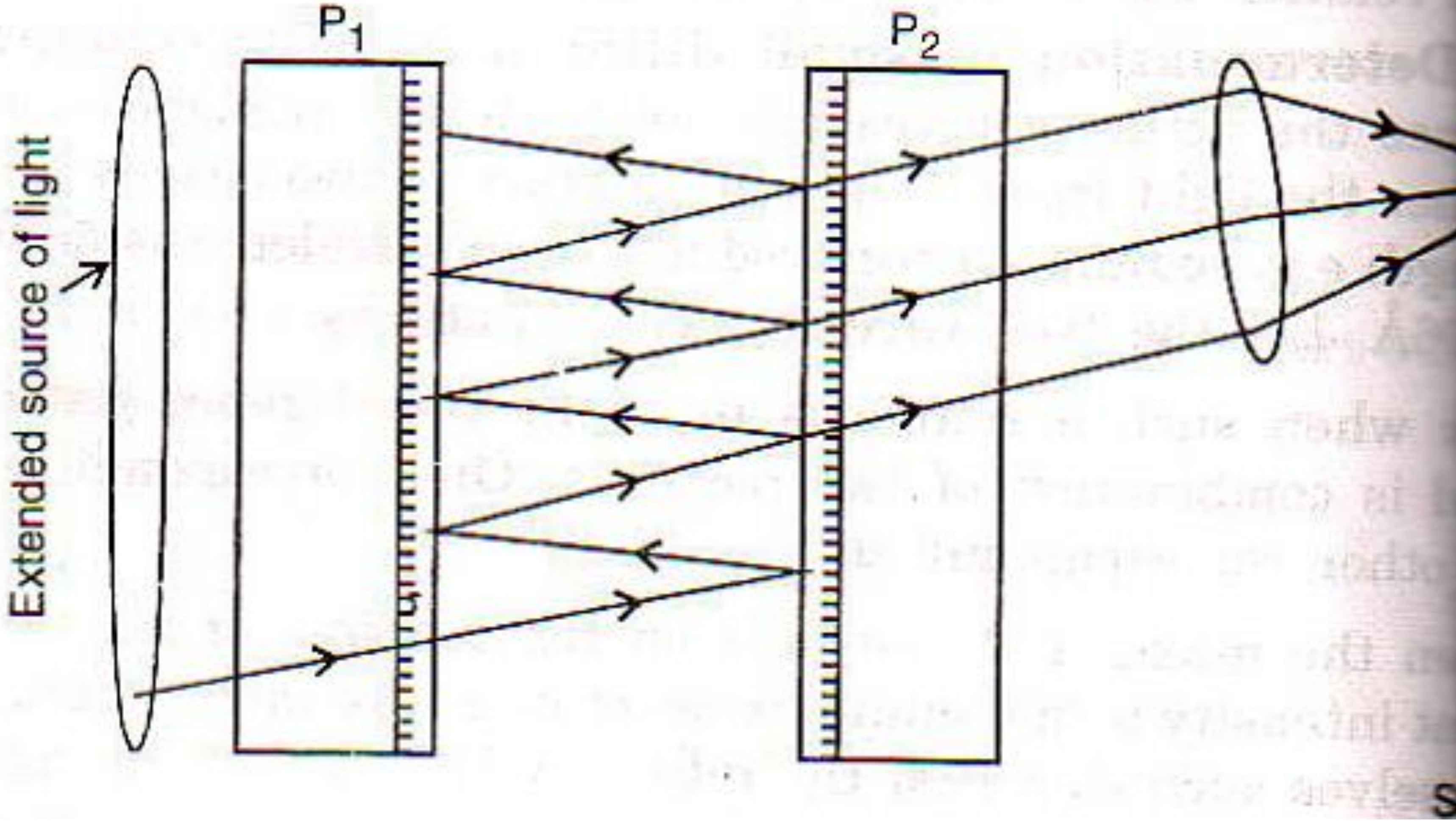
# Measurement of difference in wavelengths

- Le $\lambda 1$ and $\lambda 2$ be the two wavelengths in the incident light, $\lambda 1 > \lambda 2$
- The maxima due to wavelength $\lambda 1$ is superimposed on the maxima due to wavelength $\lambda 2$ so that the visibility in the ring is maximum. Move the movable mirror by distance d, so that visibility decreases and then becomes maximum. In such a case m1th order maximum of $\lambda 1$ wavelength overlaps on the m2th order of $\lambda 2$.

- $m\lambda_1=(m+1)\lambda_2=2d$
- $m= 2d/ \lambda_1 \quad m+1= 2d/ \lambda_2$
- $2d/ \lambda_2-1=2d/ \lambda_1 = m$
- $2d/ \lambda_2 - 2d/ \lambda_1 = 1$
- Or $\lambda_1 - \lambda_2 = \lambda_1 \lambda_2/2d \approx \lambda_1^2/2d$

# FABRY PEROT INTERFEROMETER

- **Principle.** It is based on the principle of interference by multiple reflections.
- **Construction.** It consists of two quartz plates arranged parallel to each other. These two plates enclose a thin air film between them. Multiple reflections taking place in this thin film cause the interference pattern . One plate is polished enough to make it completely reflecting while other is kept partially reflecting and partially transmitting as shown in fig in next slide .


Extended source of light
$P_1$
$P_2$
S

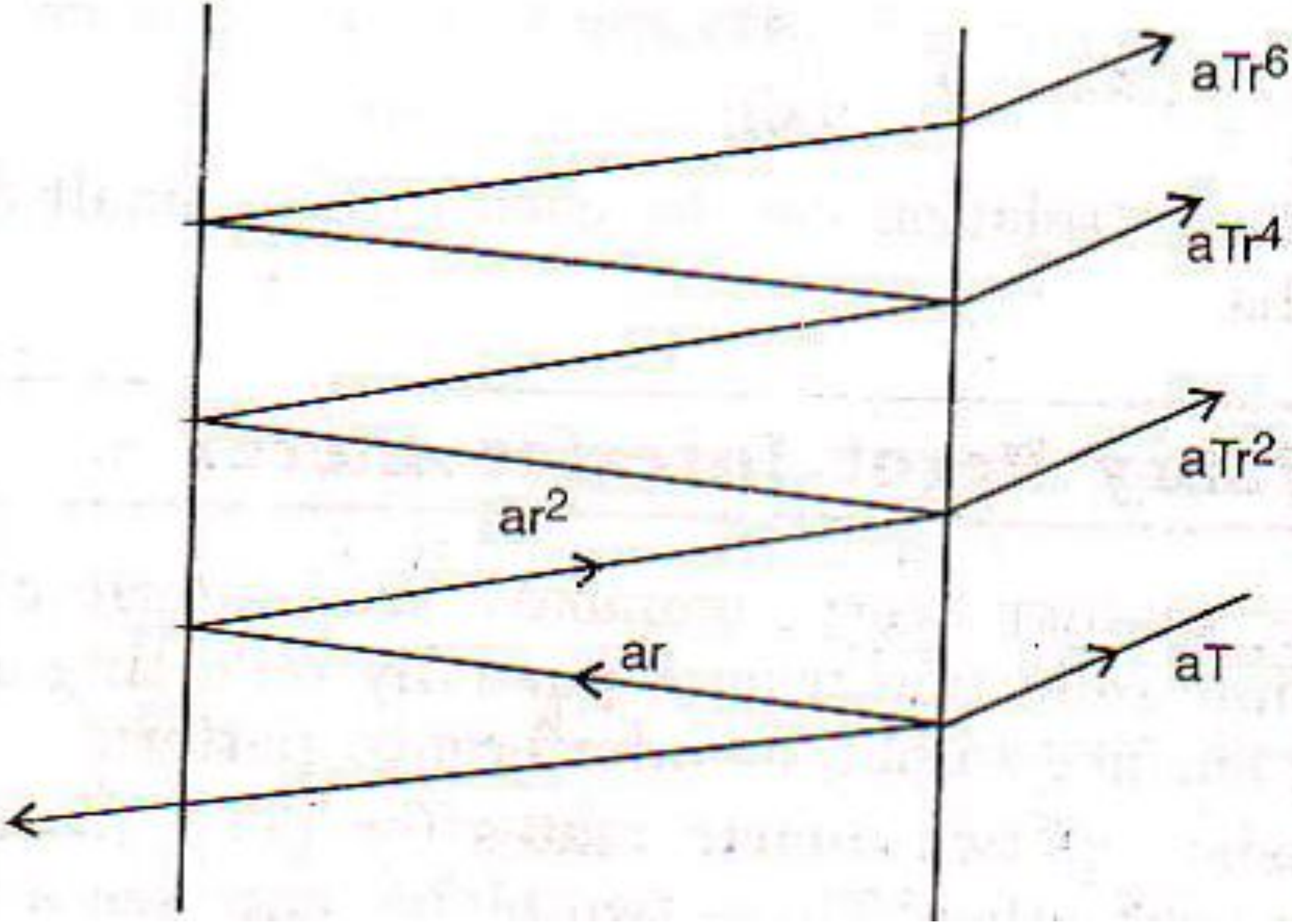
# Fabry Perot Interferometer

It uses multiple reflection from a partially reflecting and partially refracting surface to obtain interference pattern.

It consists of two quartz plates or glass plates arranged parallel to each other. These two plates encloses a thin air film between them. Multiple reflections taking place in this thin film cause the interference pattern.

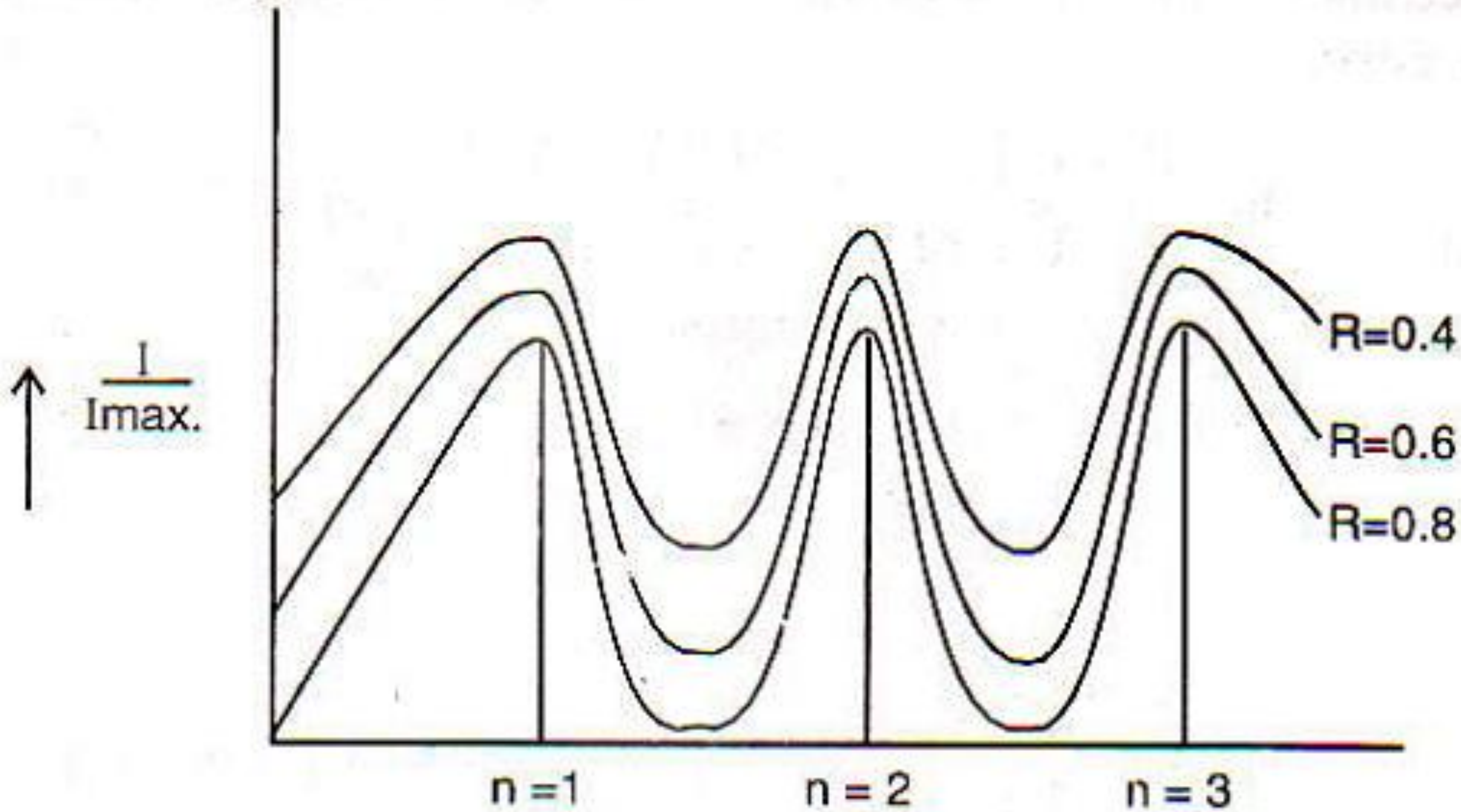
- In general one plate is kept constant and other is moved parallel to itself . The interference pattern  consists of circular alternate bright and  dark fringes . The sharpness of fringe pattern in F . P. interferometer is more than in Michelson interferometer which makes it better in resolving two closely lying wavelengths

- Intensity Distribution: Let a be the amplitude of incident wave. Then amplitude of first transmitted wave will be aT.



aTr6
aTr4
aTr2
ar2
ar
aT

- Then intensity of the resultant wave is given by
- $I = (a^2 T^2) / (1 + R^2 - 2 R \cos ?)$
- $(I_{max} / I_{min}) = ((1 + R) / (1 - R))^2$
- Fig on the next slide shows the variation of I/Imax w.r.t order of interference pattern.



I
Imax.
R=0.4
R=0.6
R=0.8
n =1
n = 2
n = 3