Phase Space

- The space in which the state of constituent particles of a system can be described in terms of position co-ordinates is called position space.
- The space in which the state of constituent particles of a system can be described in terms of momentum co-ordinates is called momentum space.
- The space in which the state of constituent particles of a system can be described in terms of position and momentum co-ordinates is called phase space.

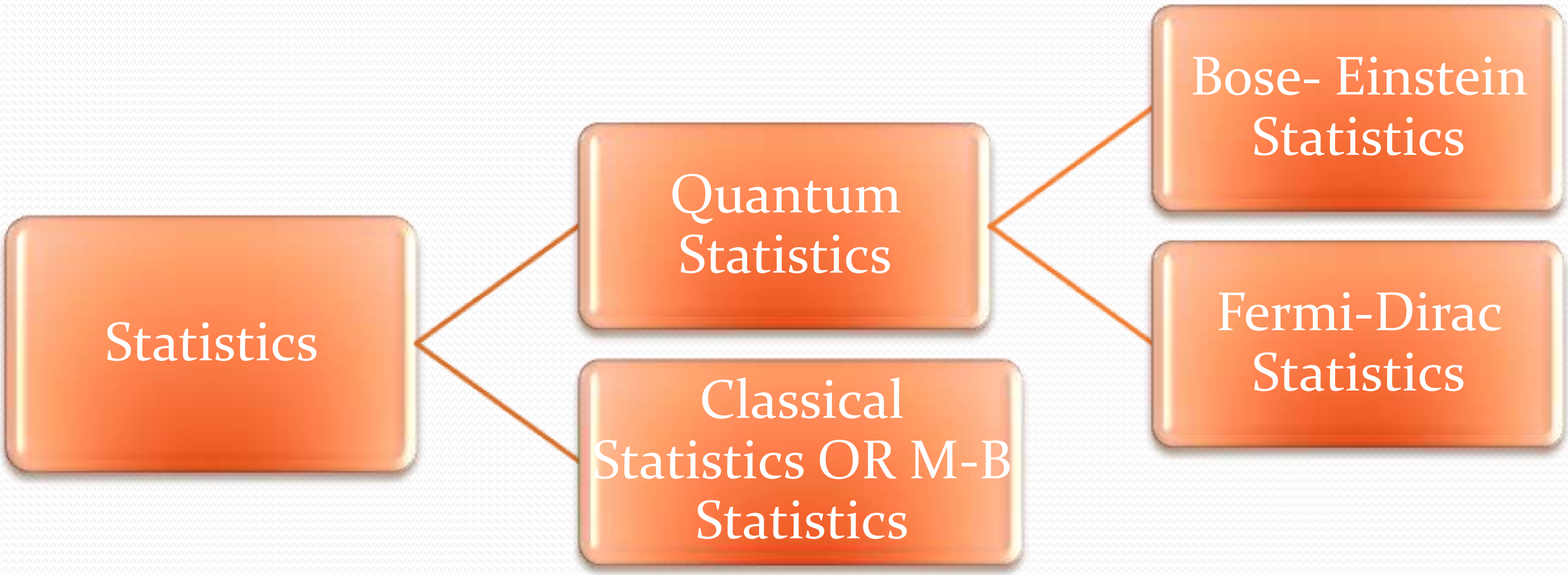
Number of the phase space cells in the momentum interval p and p+dp

$$g_p dp = \frac{\text{Available volume corresponding to momentum interval p and p+dp}}{\text{volume of each phase space cell}}$$

$$g_p dp = \frac{\iiint dx\,dy\,dz \iiint dp_x\,dp_y\,dp_z}{h_0^3}$$

$$g_p dp = \frac{v \times 4\pi p^2\,dp}{h_o^3}$$

STATISTICS


Statistics
Quantum Statistics
Classical Statistics OR M-B Statistics
Bose- Einstein Statistics
Fermi-Dirac Statistics

BASIC APPROACH IN THREE STATISTICS

- In each statistics we calculate the thermodynamic probability (W) for any given macrostate .

$$d(\ln W) = 0$$

- In any system consisting of given number n of particles having total energy u, quantities n and u are constant.

$$dn = \sum_i dn_i = 0$$

$$du = \sum_i^k \left[u_i dn_i\right] = 0$$

from these three equations we have the basic equation

which is used in all the three kinds of statistics

$$d(\ln W) = \sum \left(\alpha + \beta u_i\right) dn_i = 0$$

MAXWELL BOLTZMANN STATISTICS

Assumptions of MB statistics

1. The available volume of the phase space cell can be as small as we desire and may even approach to zero.
2. The phase space can be divided into a very large number of cells.
3. Any number of particles can occupy a phase space cell.
4. The particles of the system are distinguishable.

MAXWELL BOLTZMANN DISTRIBUTION LAW

- Consider an sample of an ideal gas consisting of n molecules and total energy u.

$$dn = \sum_{i}^{k} dn_i = 0$$

$$du = \sum_{i}^{k} n_i du_i = 0$$

- N particles are distributed in k energy intervals such that each interval is divided into cells i.e. $g_1, g_2, g_3 \ldots.. g_k$ then thermodynamic probability of equilibrium state is

MAXWELL BOLTZMANN DISTRIBUTION LAW…….

$$\mathrm{w}(\mathrm{n}_{1,} n_2, n_3, \ldots . n_k) = n! \prod_{i=1}^{i=k} \frac{(g_1)^{n_i}}{n_i!}$$

for the equilibrium state $\ln W$ is constant

therfore $d(\ln W) = 0$

using these equations we get maxwell boltzmann distribution law of energy distribution

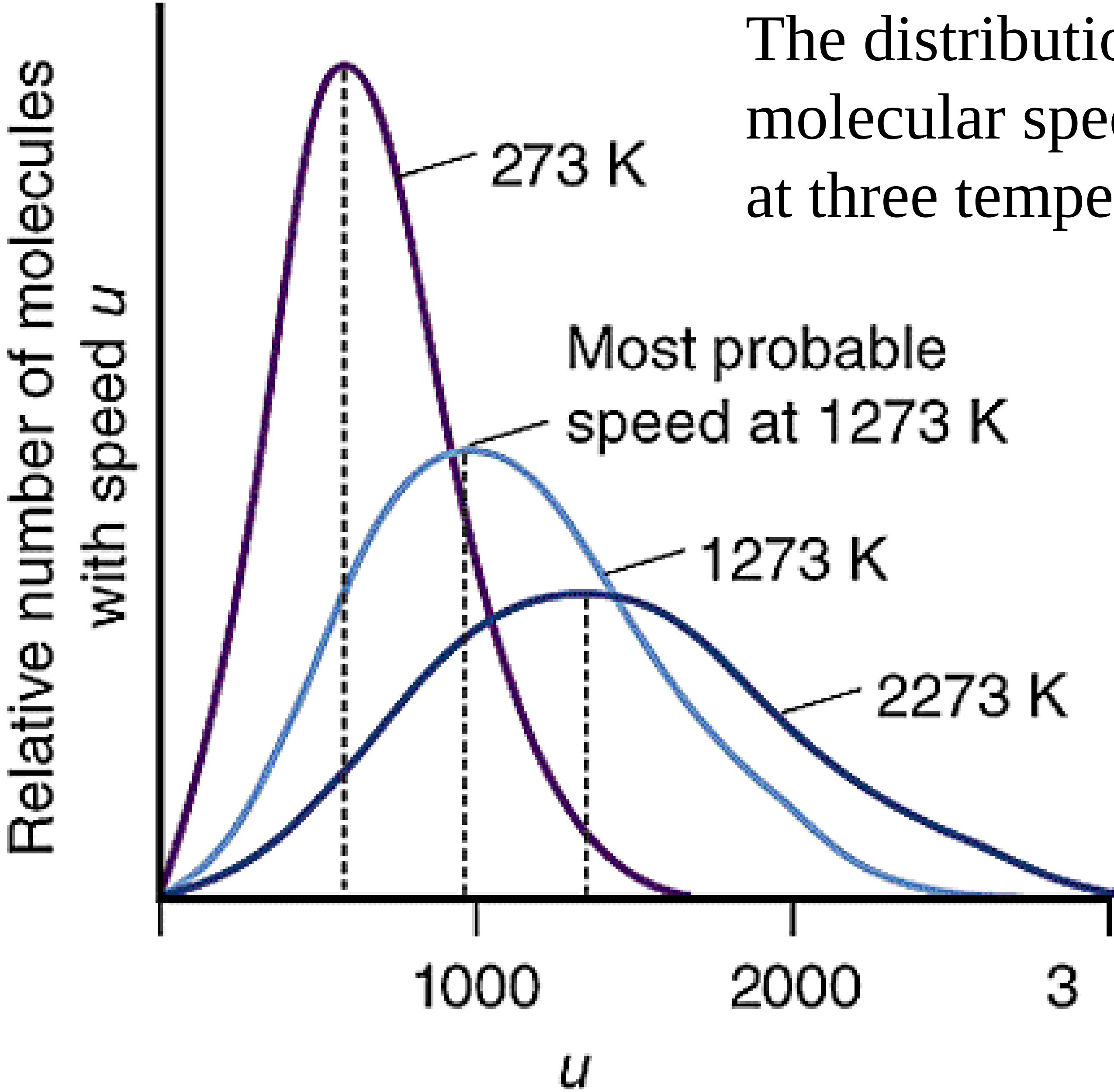
$$\mathrm{n}_{\mathrm{u}} du = \frac{2\pi n}{(\pi kT)^{\frac{3}{2}}} \sqrt{u} e^{\frac{-u}{kT}} du$$

DISTRIBUTION OF SPEEDS

$$n_v dv = \sqrt{2}\pi n\left(\frac{\mathrm{m}}{\pi \mathrm{kT}}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv$$

this expression gives the number of particles having velocity within v and v + dv for an ideal gas having n molecules at absolute temerature T

The distribution of molecular speeds for N_2 at three temperatures



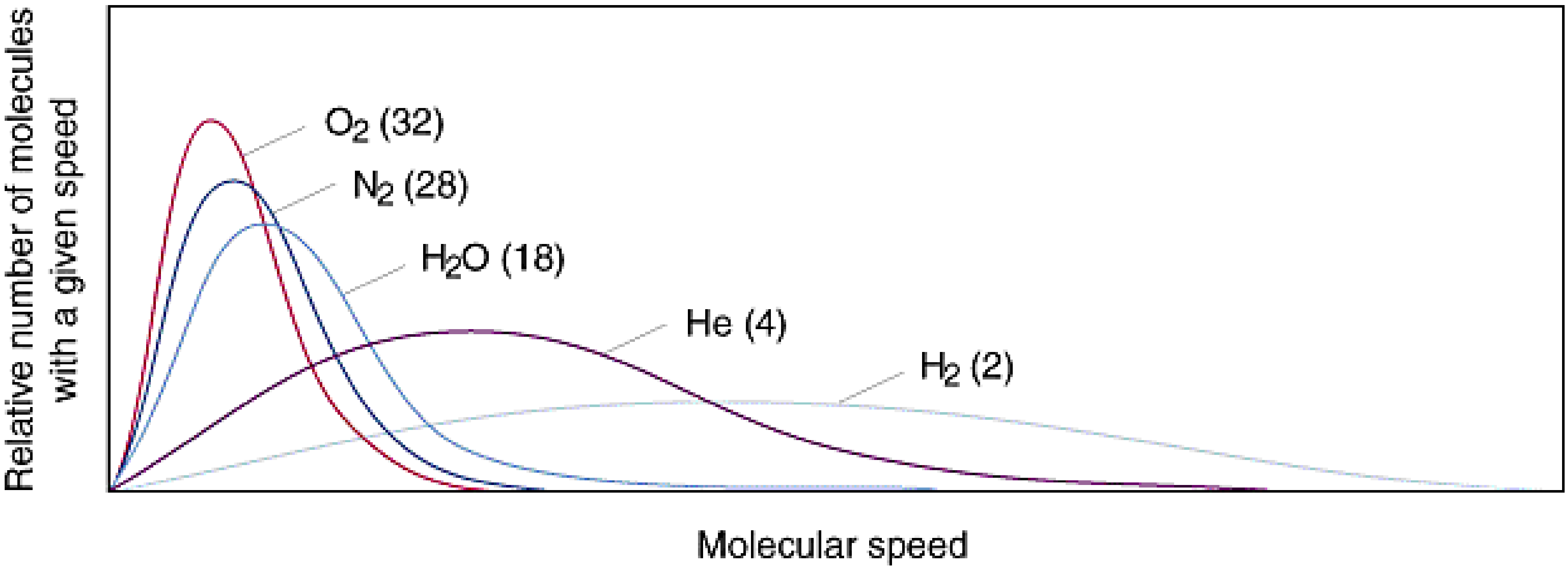
Features of the Speed Distribution

The most probable speed is at the peak of the curve.

The most probable speed increases as the temperature increases.

The distribution broadens as the temperature increases.

Relationship between molar mass and molecular speed



Features of the Speed Distribution

The most probable speed increases as the molecular mass decreases.

The distribution broadens as the molecular mass decreases.

MOST PROBABLE SPEED

- It is defined as the speed possessed by the maximum number of molecules in a sample at a given temperature

$$v_m = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$$

Average speed

- It is defined as the average of the speed of all the gas molecules lies between v and v + dv

$$v_{av} = 1.59\sqrt{\frac{kT}{m}}$$

ROOT MEAN SQUARE SPEED

- It is defined as the square root of means of the squares of the speeds of all the molecules

$$v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$$

Relation between v_m, v_{av} and v_{rms}

$$v_m : v_{av} : v_{rms} :: 1.41 : 1.59 : 1.73$$

$$v_m < v_{av} < v_{rms}$$

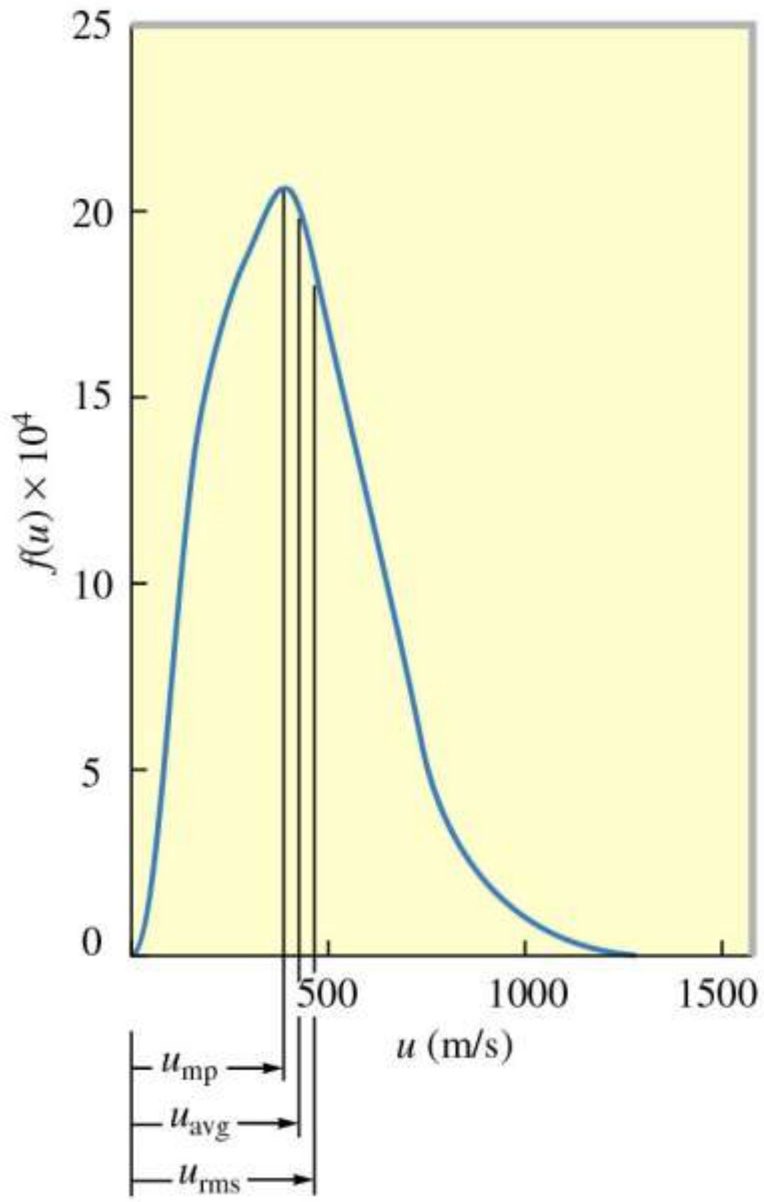
The Three Measures of the Speed of a Typical Particle

$$u_{rms} = \sqrt{\frac{3RT}{M}}$$

$$u_{mp} = \sqrt{\frac{2RT}{M}}$$

$$u_{avg} = \bar{u} = \sqrt{\frac{8RT}{\pi M}}$$

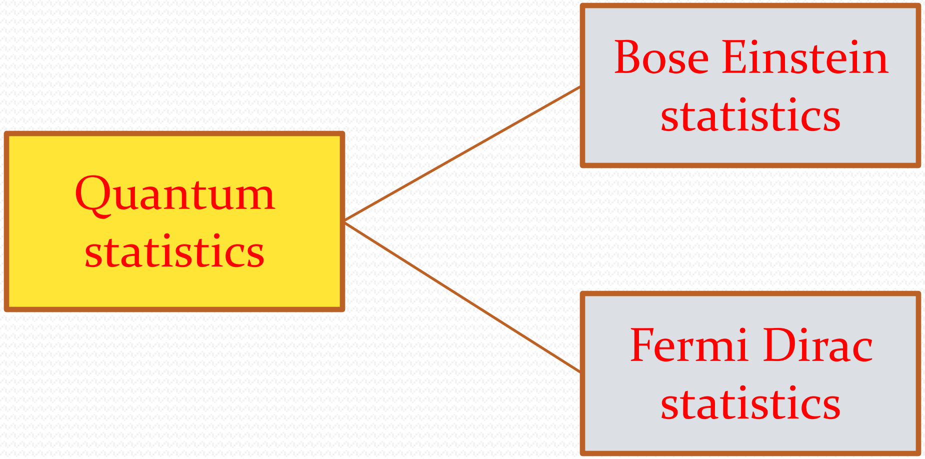
Various Ways to Summarize the 'Mean' Speed.



Limitations of MB statistics- Birth of Quantum statistics

- Maxwell Boltzmann statistics is able to explain the distribution of energy among the molecules of a gas.
- But system of photons and electrons are subjected to certain constraints which are not applicable to gas systems.
- This leads to the origin of quantum statistics which is based on the concept of quantization of energy.

Quantum statistics



Bose Einstein statistics

Basic assumptions

- The particles of the system are indistinguishable and identical.
- Available volume of the phase space cell cannot be less than h3 , where h is Planck's constant.
- Any number of particles can occupy a phase space cell.
- The number of phase space cells is comparable with the number of particles .
- The particles under consideration do not obey Pauli's exclusion principle.

BOSONS

- Bosons are the particles of a system whose energy spectra can be explained on the basis of BE statistics
- Bosons do not obey Pauli's exclusion principle.
- Boson have integral spin.
- Examples : photons, K and pi mesons etc.

Fermi Dirac statistics

Basic assumptions

- The particles of the system are indistinguishable and identical.
- Available volume of the phase space cell cannot be less than h3 , where h is Planck's constant.
- A phase space cell cannot more than one particle.
- The number of phase space cells is large as compared with the number of particles .
- The particles under consideration obey Pauli's exclusion principle.

FERMIONS

- fermions are the particles of a system whose energy spectra can be explained on the basis of FD statistics
- fermions obey Pauli's exclusion principle.
- fermions have half integral spin.
- Examples : electrons , positrons, neutrons etc.

Size of phase space cell in quantum statistics

According to uncertainty principle

$\Delta x \, \Delta p_x \geq h$

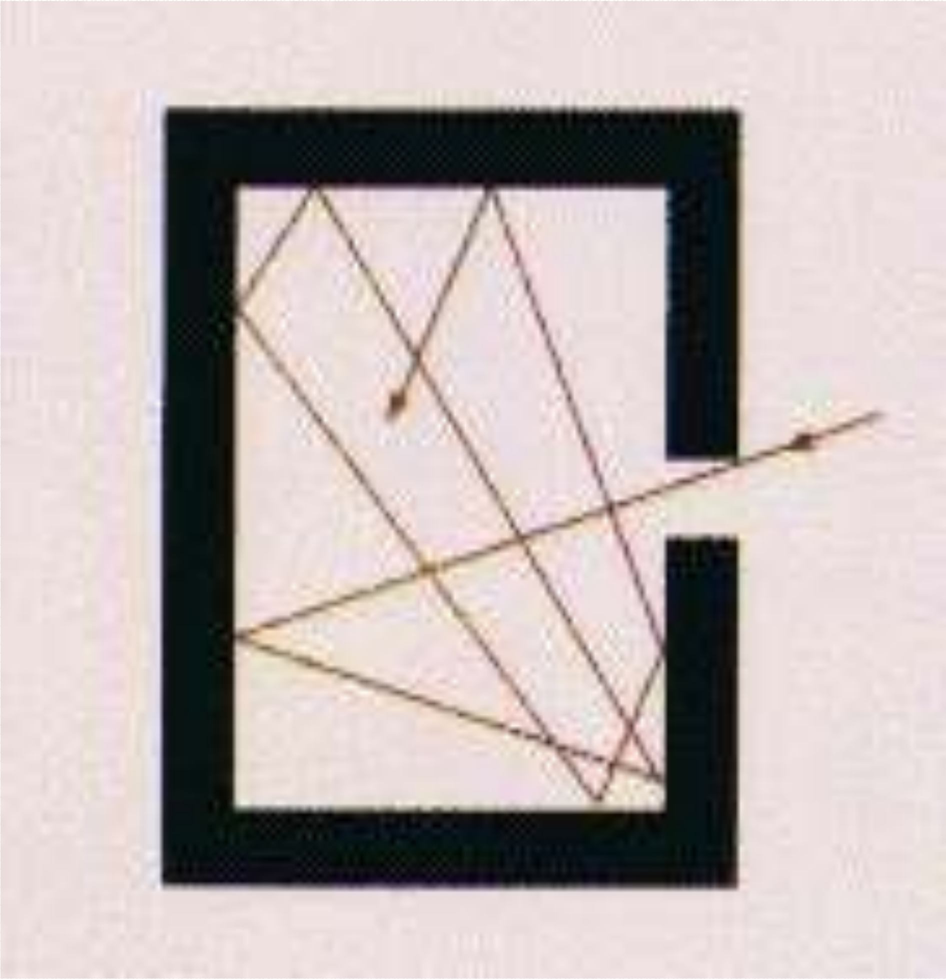
therefore

Available volume $\Delta x \, \Delta y \, \Delta z \, \Delta p_x \, \Delta p_y \, \Delta p_z = h^3$

that is size of the phase space cell cannot be less than h

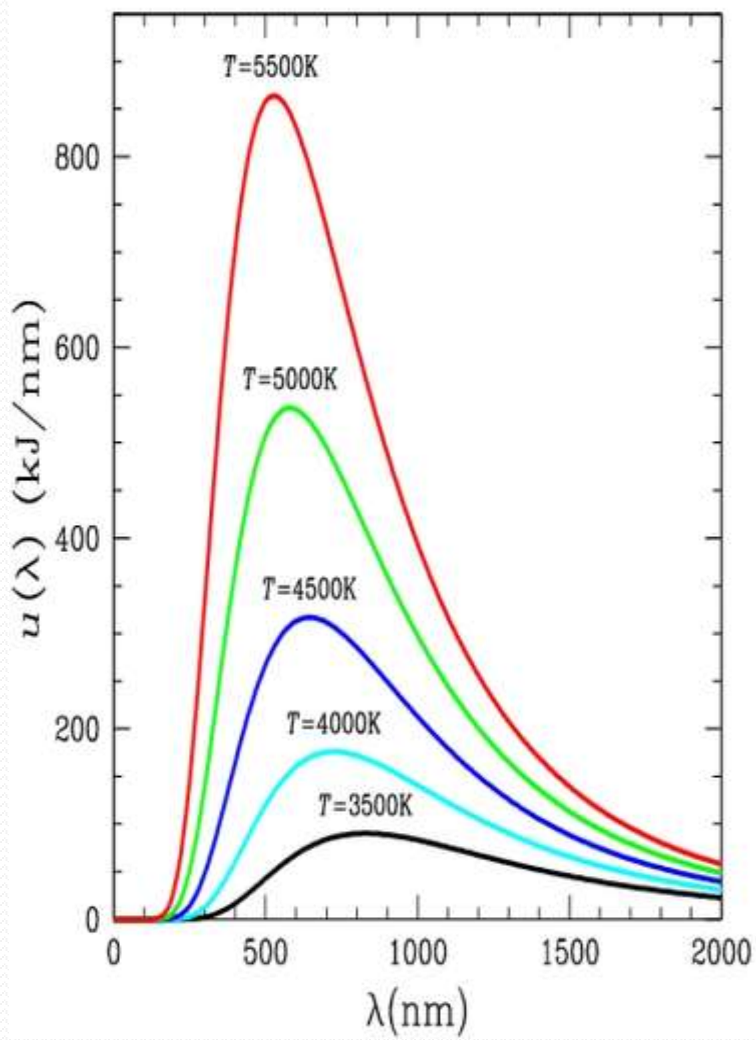
Definition of a black body

A black body is an ideal body which allows the whole of the incident radiation to pass into itself (without reflecting the energy) and absorbs within itself this whole incident radiation (without passing on the energy). This propety is valid for radiation corresponding to all wavelengths and to all angels of incidence. Therefore, the black body is an ideal absorber of incident radaition.



ENERGY DISTRIBUTION IN THE SPECTRUM OF BLACK BODY RADIATIONS

- Emissive power of the black body is greater at higher temperatures.
- Amount of energy emitted per second increases for all wavelengths with increase of temperature of black body.
- Amount of energy emitted per second is higher for intermediate wavelength.
- Corresponding to every temperature of the black body there is a wavelength for which the emissive power is highest.
- The wavelength corresponding to maximum emission shifts towards shorter values as the temperature of the black body increases.



Black-Body Radiation Laws

Planck Law

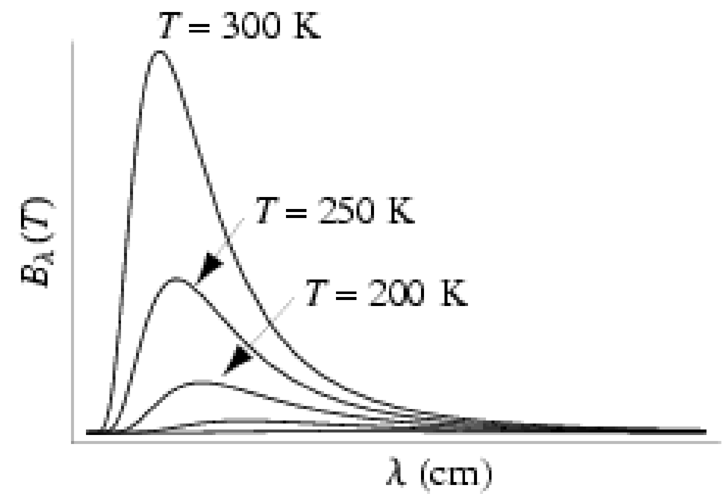
We have two forms. As a function of wavelength.

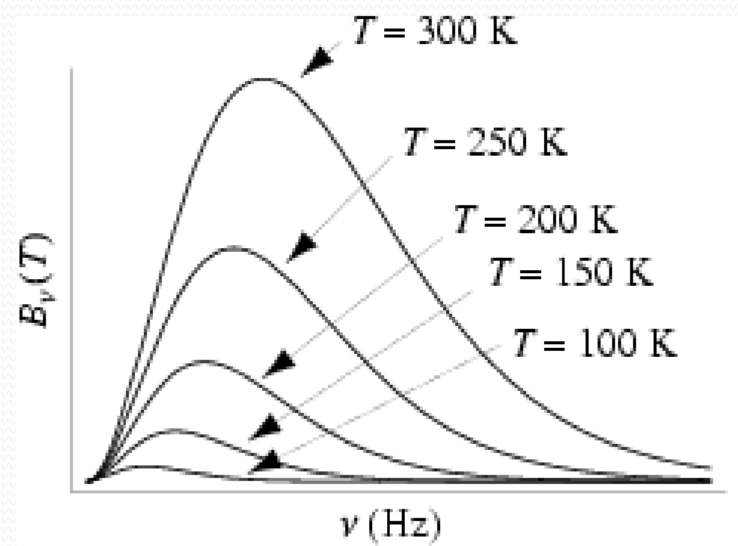
$$E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{kT}} - 1} d\lambda$$

And as a function of frequency

$$\mathrm{E}_\nu \, \mathrm{d}\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}} - 1} d\nu$$

The Planck Law gives a distribution that peaks at a certain wavelength, the peak shifts to shorter wavelengths for higher temperatures, and the area under the curve grows rapidly with increasing temperature.





Wein's displacement law

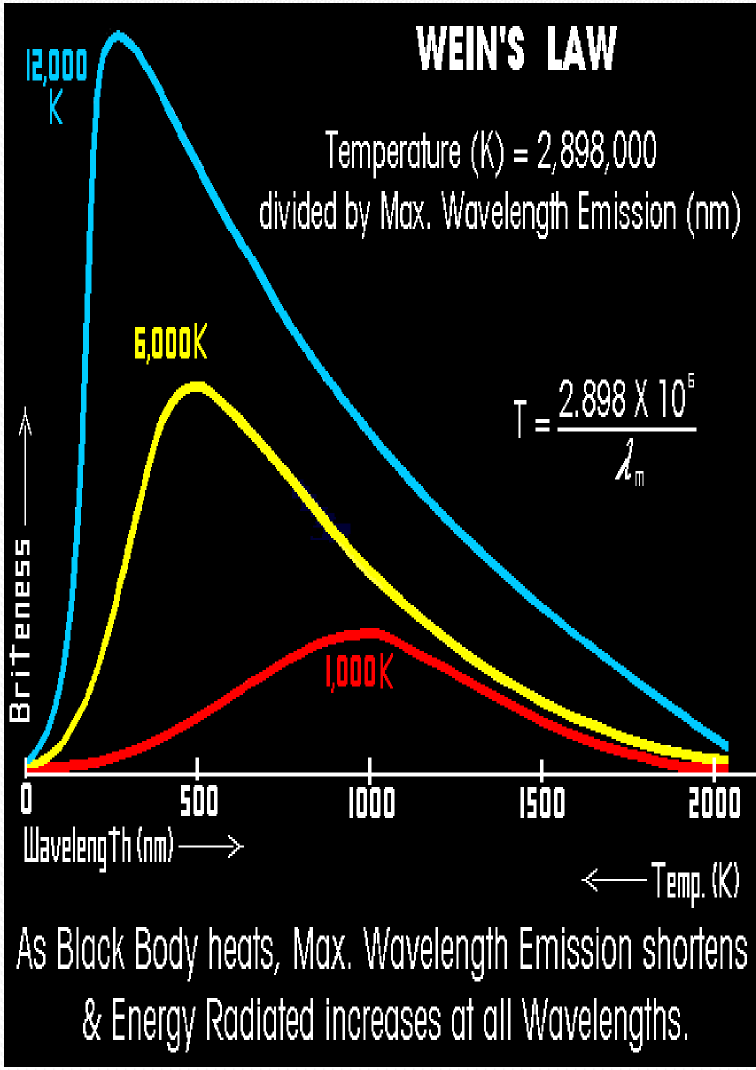
- Wavelength λ_m of the radiations corresponding to maximum emissive power of the black body varies inversely as the absolute temperature T.

$$\lambda_m \propto \frac{1}{T}$$

$$\lambda_m T = \text{a constant}$$

- The wavelength corresponding to maximum emission shifts towards shorter values as the temperature of the black body increases.

- This law tells us as we heat an object up, its color changes from red to orange to white hot.



Rayleigh jean’s law

- Energy density of the radiations corresponding to the wavelength λ is

$$E_{\lambda} d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

- It could explain the experimentally observed energy distribution only for long wavelengths.
- Rayleigh jean’s law is a special case of Planck's law for longer wavelengths.

Stefan’s law

- Stefan’s law states that the amount of energy of the radiations emitted from a perfectly black body per second per unit area is directly proportional to the fourth power of absolute temperature.

$$\text{i.e.} \quad E \propto T^4$$

- It is valid for whole range of wavelengths .
- It is a special case of Planck's law.

Summary

- A black body is a theoretical object that absorbs 100% of the radiation that hits it. Therefore it reflects no radiation and appears perfectly black.
- Roughly we can say that the stars radiate like blackbody radiators. This is important because it means that we can use the theory for blackbody radiators to infer things about stars.
- At a particular temperature the black body would emit the maximum amount of energy possible for that temperature.
- Blackbody radiation does not depend on the type of object emitting it. Entire spectrum of blackbody radiation depends on only one parameter, the temperature, T.

MB statistics as special case of BE and FD statistics

$$\text{FD distribution} \Rightarrow \mathrm{n_u du} = \frac{\mathrm{n_u}}{e^{\alpha} e^{\frac{u}{kT}} + 1} \mathrm{du}$$

$$\text{BE distribution} \Rightarrow \mathrm{n_u du} = \frac{\mathrm{g}}{e^{\alpha} e^{\frac{u}{kT}} - 1} \mathrm{du}$$

$$\text{MB distribution} \Rightarrow \mathrm{n_u du} = \frac{\mathrm{g_u}}{e^{\alpha} e^{\frac{u}{kT}}} \mathrm{du}$$

MB statistics as special case of BE and FD statistics…….

For low occupation index

$$\frac{n_u}{g_u} << 1$$

for FD statistics: $\left(e^{\alpha} e^{\frac{u}{kT}} + 1\right) >> 1$

for BE statistics: $\left(e^{\alpha} e^{\frac{u}{kT}} - 1\right) >> 1$

neglecting +1 and -1 , FD and BE statistics reduce to MB statistics